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ON AN INTEGRAL RELATION BETWEEN THE SLOPES OF THE BRANCHES OF PORTER GRAPHS*

By Henry A. Blair

SYNOPSIS

A relation was found to exist between the slopes of the branches of the Porter Graphs for a number of colors throughout the spectrum such that the slopes of the branches of each graph can be expressed as integral multiples of a common factor.

Critical frequency measurements on the tactile sensoria which show a similar relation are discussed in corroboration.

The relation is interpreted as indicating that the physiological effect measured by the critical frequency for any particular color is made up of equal elements which start off in groups from different intensity thresholds. The possibility that these elements denote the action of single end organs or associated groups of end organs is discussed in the light of the measurements made by Adrian and Zotterman on the frequency of the nerve impulses due to the stimulation of single end organs. The interpretation is conformable to the "all or none" law.

It has been confirmed by many observers that the Ferry-Porter[1] law,

$$1/D = K \log I + K'$$

represents the relation between the intensity I of a light stimulus and the critical frequency of flicker, $1/D$, where D is the duration of an individual stimulus when a succession of like stimuli are frequent enough to give a just continuous sensation.

Recently it has been shown by Allen[2] that, instead of there being only two branches in the graphical representation of the law on a semi-logarithmic scale, there are for many colors at least four or five

* The graphical representations of the Ferry-Porter law.

[1] Ferry Am. Jour. Sci., *3*, p. 44; 1892.
Porter, Proc. Roy. Soc., *70*, p. 313; 1901.

[2] J. O. S. A. & R. S. I., *13*, p. 383; Oct. 1926.

branches. Allen has named the branches by means of the Greek letters in alphabetical succession, the first branch for the lowest intensity being called the α branch and the corresponding intensity range the α intensity, (v. fig. 2). He has also called the phenomenon represented by the Ferry-Porter law the Porter effect, and the graph for any particular color a Porter graph.

A Porter graph has the form shown by the full line in Fig. 1. That is, it is made up of line segments whose slopes may increase or decrease on going from the one to the other. It is evident that it may be considered to be built up of the dotted components, and it seems not improbable

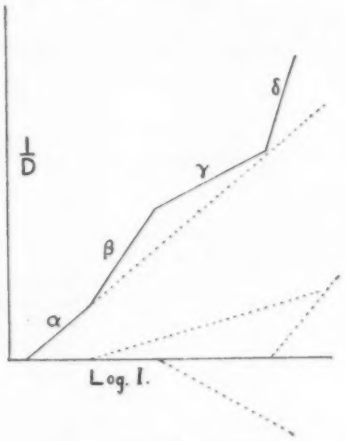

FIG. 1

that the graph does actually represent the summation of a number of effects with different thresholds, particularly since $1/D$ is considered to be a measure of the luminous sensation.

After obtaining the α branch which represents the initial effect above the lowest threshold and which may be represented by the equation,

$$n_1 = k_1 \log I - a_1$$

it is seen that the β branch may be due to the entrance of a new effect,

$$n_2 = k_2 \log I - a_2$$

which, starting at a new threshold, is added to the initial effect to give the complete effect shown by the β branch.

After crossing the β interval of intensity the γ branch, whose slope is smaller in this case than that of the β branch, may be considered to be due to the entering of a new component of the form,

$$n' = k' \log I + a'$$

and in a similar way may all the branches be formed whatever their number. The particular type of equation used for each branch is determined by the nature of the change of slope.

When $n = 0$ in the equations of the type,

$$n = k \log I - a$$

$$a = k \log I_0$$

where I_0 is the threshold intensity for this particular effect. Therefore these equations may be written in the form,

$$n = k \log I - k \log I_0$$

or

$$n = k \log I/I_0$$

Similarly the equations of the form,

$$n = -k \log I + a$$

may be written,

$$n = k \log I_0/I$$

Hence the equation of the complete Porter graph may be written,

$$1/D = n_1 + n_2 + \cdots + n' + n'' + \cdots$$
$$= k_1 \log I/I_1 + \cdots + k' \log I'/I + \cdots$$

where the subscript and prime values of I represent the threshold intensities for the various effects.

This last equation may be written,

$$1/D = \log (I^{k_1+k_2+\cdots}/I^{k'+k''+\cdots}) + A$$

where A represents the threshold terms, or

$$1/D = \log I^{k_1+k_2+\cdots-k'-k''-\cdots} + A$$

which may be written,

$$1/D = C \log I + A$$

where C and A take on new values for each branch. Also C will be the slope of any branch when I is plotted on a logarithmic scale.

In regard to the slopes two cases may exist:

(1) The k's may be all different and have no simple relation to each other, in which case the different values of C will have no simple relation.

(2) The k's may have some simple relation, even the simplest which suggests itself, i.e., they may be all integral multiples of a common factor.

If the second case be true the equation of any branch may be written,

$$1/D = c(m-n) \log I + A$$

where c is the common factor or elementary constant and m and n are integers.

This possible relation gives the three main conditions, $m-n=0$, $m-n=$a positive integer and $m-n=$a negative integer. The case where $m-n=0$, i.e., where $1/D$ is constant for a range of intensities and the branch of the Porter graph is horizontal, has been found to occur in vision.[3] Measurements in vision usually give a relation in which $1/D$ increases as the intensity increases so these measurements may be tested for the case $m-n=$a positive integer. The case in which $m-n$ might be a negative integer has not been found in vision, but measurements on the taste organs by a similar method[4] gave always a relation in which $1/D$ decreased as the intensity increased.

The first set of Porter graphs examined for this relation was Allen's[2] which is reproduced here in Figs. 2 and 3. As is usual with spectral measurements the intensity ranges are not the same for the different wave lengths and the relation is not known. Therefore these graphs could not be used to determine whether the elementary constant, if it existed, applied to all colors. However, each graph in turn was tested to see if its slopes showed the relation $c(m-n)$.

As will be noted most of the lines are quite accurately determined experimentally and these were the only ones whose slopes were measured. Even in these cases, however, there remains often a certain amount of choice in the way in which the lines may be drawn, but it is not to be expected that the error in these cases will be very large, especially with the longer lines. Apart from this the actual measurements were made to two or three parts in one thousand in general, and since the vertical projection of each line was divided by the hori-

[3] Ives, Phil. Mag., *24*, p. 352; 1912.
Allen, Phil. Mag., *38*, p. 81; 1919.

[4] Allen and Weinberg, Quar. Jour. Exp. Physiol., *14*, p. 351; 1924.

zontal projection to get the slope, it is unlikely that the error in the slope will exceed .005 in any case as far as measurement is concerned. It is difficult to estimate the error due to latitude in drawing the lines but probably the slopes can be considered to be accurate to at least the

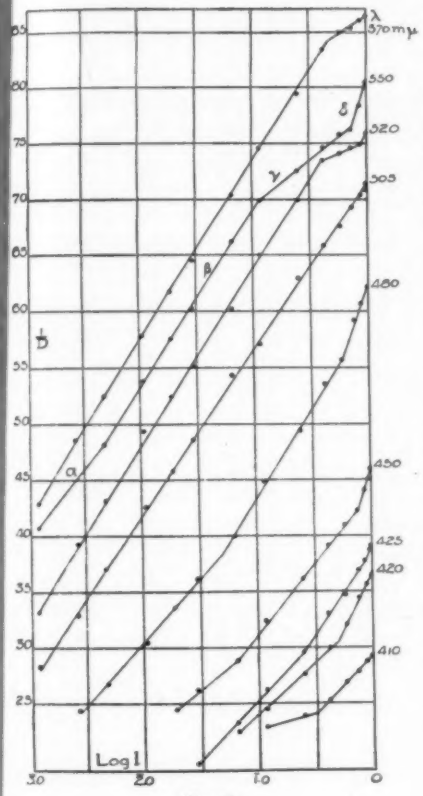

FIG. 2

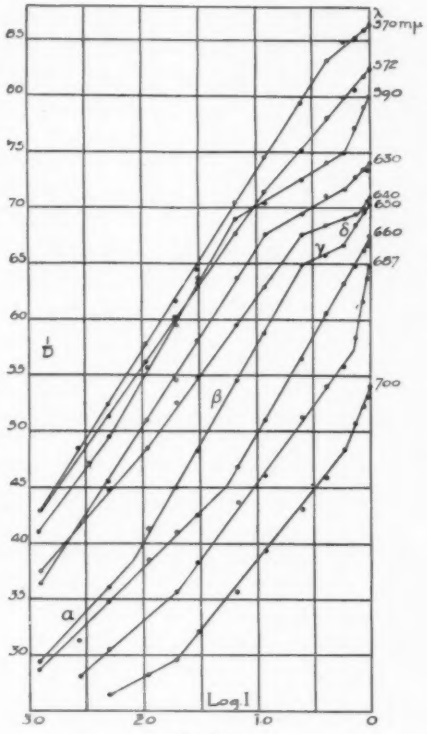

FIG. 3

second place of decimals in most cases and frequently with no great error in the third.

In Table 1 are given the slopes of all the well determined branches for each wave length. At the top of each column is given the probable factor c of the slopes of the column. It was usually found by taking the difference between the two most nearly equal slopes for each color and dividing this difference by integers until a number was obtained which was most nearly a factor of all the slopes. The row marked

$c(m-n)$ gives the product of the elementary factor c and the integer by which it must be multiplied to give the measured slope, while the row $(m-n)$ gives the integer.

TABLE 1

Branch	Wave length	410 mμ	420 mμ	425 mμ	450 mμ	480 mμ	505 mμ	520 mμ	550 mμ
	c	.094	.099	.103	.0877	.092	.096	.102	.0853
α	Slope	.282	.891	1.031	.789	1.104	1.445	1.538	
	$c(m-n)$	.282	.891	1.030	.789	1.104	1.440	1.530	
	$m-n$	3	9	10	9	12	15	15	
	Slope	1.033	1.985	1.444	1.224	1.569			1.535
β	$c(m-n)$	1.034	1.980	1.442	1.227	1.564		as α	1.535
	$m-n$	11	20	14	14	17			18
	Slope							.408	.782
γ	$c(m-n)$	as β		as β	as β	as β		.408	.767
	$m-n$							4	9
	Slope								2.900
δ	$c(m-n)$								2.900
	$m-n$								34

	Wave length	570 mμ	572 mμ	590 mμ	630 mμ	640 mμ	660 mμ	687 mμ	700 mμ
	c	.020	.061	.123	.051	.124	.122	.124	.087
	Slope	1.543	1.403	1.719	1.523	1.116	.973	.872	.525
α	$c(m-n)$	1.540	1.403	1.722	1.530	1.116	.976	.868	.522
	$m-n$	77	23	14	30	9	8	7	6
	Slope					1.363		1.359	
β	$c(m-n)$	as α	as α	as α	as α	1.368	as α	1.364	
	$m-n$					11		11	
	Slope		1.159	.614	.566	.375	1.096		2.435
γ	$c(m-n)$	as α	1.159	.615	.561	.372	1.098		2.436
	$m-n$		19	5	11	3	9		28
	Slope	.782		2.090	1.070				
δ	$c(m-n)$	.780	as γ	2.091	1.071				
	$m-n$	39		17	21				

It will be readily observed on examining the table that in every case a factor was found such that $c(m-n)$ was as nearly equal to the measured value of the slope as could be expected considering the possible accuracy in measurement. Moreover the average value of the factor is about

.090. Since there are fifteen cases with either two or three slopes it will be seen that the probability of a factor of this size dividing into all the slopes of each graph, as well as it did in every instance, is exceedingly small unless the slopes are related in the way which has been supposed. Hence there is good ground for the belief that the slopes of the various branches of each Porter graph are integral multiples of a common factor. Whether or not the same factor would apply to the graphs for all wave lengths, if they were all plotted on the same scale, can only be determined by further experiment using known intensity ranges so that a comparison can be made.

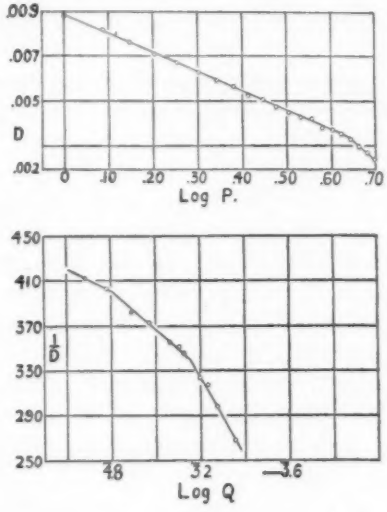

FIG. 4

The corresponding measurements on the organs of taste,[4] previously mentioned, give four normal graphs corresponding to the four fundamental tastes. These graphs, a typical example of which is given in Fig. 4, (lower) are not so well defined experimentally as those in vision but measurements upon them give good indication that their slopes are related in a similar way to those which have been discussed here. Furthermore, a factor of the same numerical value applies to all four of the graphs. By analogy there is a possibility that the same thing will occur in vision, i.e., that the same numerical factor will apply to the graphs of all colors when they are plotted on equal intensity scales.

Measurements similar to those discussed in taste and vision have been made in Allen's laboratory on the tactile sensoria.[5] These were obtained by observing the point of fusion of the sensations due to intermittent pulses of air. It was found that they gave the relation,

$$D = -K \log P + K!$$

where D is the duration of an individual stimulus at the point of fusion and P is the pressure of the air. Allen called this effect and the corresponding effect in taste Porter effects also, on account of their similarity to this phenomenon in vision.

A tactile Porter graph is given here in Fig. 4 (upper). It will be seen that it is very well determined experimentally especially as regards the α branch.

Since the touch graphs are composed of line segments similar to those in vision and taste it was expected that a similar relation would exist between the slopes of the branches. The data of the two papers[5] were plotted on the same semi-logarithmic scale so that the graphs for the three persons who made the measurements could be compared. The slopes of all the well determined branches are given in Table 2. The graphs are lettered A, B, etc., as they were originally and the branches are named, α, β, etc., starting with the lowest pressure range. The name of the person who made the measurements is given in each case along with information as to where the data and graphs were published. As in table 1, $c(m-n)$ is the product of the supposed factor and the integer which most nearly gives the measured slope. In this case the difference between the measured slope and $c(m-n)$ is also given.

After measuring all the slopes a factor was sought first for Hollenberg's graphs by taking the difference between the two most nearly equal slopes, A, α, and B, α, which is .013. This was found to divide as well as could be expected into all Hollenberg's slopes, which was not particularly surprising. It was very surprising, however, to find that the same factor applied equally as well to Allen's and Weinberg's graphs. The same apparatus was used in all cases but Weinberg's measurements were made on the lip while Allen's and Hollenberg's were made on the finger. This is either a remarkable coincidence or indicates a great similarity in the action of the sensory nerves in normal individuals.

[5] Allen and Hollenberg, Quar. Jour. Exp. Physiol., *14*, No. 14; 1924.
Allen and Weinberg, Quar. Jour. Exp. Physiol., *15*, Nos. 3 and 4; 1925.

The factor .013 is not very large compared to the possible errors in measurements especially with the short lines. The long lines which comprise the α branches are somewhat better determined than those in

TABLE 2

Hollenberg's Graphs
Quar. Jour. Exp. Physiol. *14*, Data, p. 335, Graph p. 359; 1924.

Graph	*A*	*B*		*C*		*D*	*E*
Branch	α	α	β	α	β	α	α
Slope	.911	.924	2.538	.858	1.555	.793	.713
$c(m-n)$	.910	.923	2.535	.858	1.560	.793	.715
Diff.	.001	.001	.003	.000	.005	.000	.002
$m-n$	70	71	195	66	120	61	55

Data p. 364, Graphs p. 363

Graph	*A*		*B*	*C*		*D*	*E*
Branch	α	β		α	β	α	α
Slope	.934	2.463	as C	.880	1.534	.852	.846
$c(m-n)$	.936	2.462	above	.884	1.534	.858	.845
Diff.	.002	.001		.004	.000	.006	.001
$m-n$	72	174		68	108	66	65

Allen's Graphs,
Same Journal, Data p. 355, Graph Fig. 6, p. 368.

Graph	*A*	
Branch	α	β
Slope	1.041	2.679
$c(m-n)$	1.040	2.678
Diff.	.001	.001
$m-n$	80	208

Weinberg's Graphs,
Quar. Jour. Exp. Physiol., *15*, 1925. Data p. 379, Graph p. 381.

Graph	*A*			*B*		
Branch	α	β	γ	α	β	
Slope	.912	.638	.193	.104	.746	
$c(m-n)$	.910	.637	.195	.104	.741	
Diff.	.002	.001	.002	.000	.005	
$m-n$	70	49	15	8	57	

vision. The error in measuring the slopes should not exceed .002 in these cases but probably the slopes of the short lines cannot be measured so closely. The error due to the possible latitude in drawing the lines

is very small with the α branches but may be appreciable with the β branches. It was, therefore, necessary to calculate the probability of getting the results obtained on the assumption that the slopes were actually unrelated in order to obtain a measure of the likelihood of their being related.

In Table 3 are given the numbers of occurrences of the various differences between the measured slope and $c(m-n)$ along with the probable number of these differences if the slopes were unrelated. Since the signs of the differences were not considered the probability of a difference other than zero is in any one division two-thirteenths while the probability of a zero difference is one-thirteenth.

TABLE 3

Difference	Prob. in any one division	Prob. number of these	Actual number
.000	1/13	20/13	4
.001	2/13	40/13	7
.002	"	"	4
.003	"	"	1
.004	"	"	1
.005	"	"	2
.006	"	"	1

It will be observed that the actual number of differences exceeds the probable number for the small differences while the probable number exceeds the actual number for large differences. This indicates that the slopes are not unrelated but are multiples of .013 as supposed. Furthermore the probability of the occurrence together of the differences up to .002 as they were obtained would be only one in seven thousand if the slopes were unrelated. Since these differences are unavoidable even with the best determined lines it can be concluded fairly confidently that the slopes of all the branches of all the graphs are multiples of a common factor.

These results are given here only as confirmation of the results in vision. This is considered permissible as a similarity is to be expected in all sensory nerve actions and especially since Allen has shown that several of these conform to a logarithmic relation similar to the Ferry-Porter law.

Returning now to the case of vision and letting the physiological effect which is measured by $1/D$ be represented by S, it appears that for any particular color the value of S depends on the stimulus in such

a way that it is equal to the sum of an integral number of elements all of the same kind and of the form,

$$n = \pm c \log I$$

These elementary effects start off in groups from several different thresholds fairly widely separated.

It would seem that the different thresholds may exist so that light intensity values may be better discriminated, for since

$$dn/dI = c/I$$

n will change very slowly with I as I becomes large compared with c. Hence S will change very slowly with increase of intensity after a certain stage determined by the number of its components n, and if the values of S give visual discrimination, this discrimination will become poor with high intensities. Increasing the number of components of S while having only one threshold gives very good discrimination with low intensities and moves the stage at which discrimination becomes poor toward the high intensities. Similar discrimination with all intensities can however be obtained by having a number of different thresholds at which new elements of S may enter, and this last condition is probably the most satisfactory, in general, to good vision.

Since it is almost inevitable that the Porter effect is a manifestation of nerve action it is necessary to interpret, if possible, the existence of the elements in the Porter effect in accord with existing knowledge of nerve phenomena.

The recent work of Adrian[6] and Adrian and Zotterman[7] on the measurement of impulses produced by sensory nerve endings indicates very clearly that the "all or none" law applies to the sensory nerves as far as electrical impulses are concerned at least. They found that the impulses along a particular nerve fibre were all of the same size but that the frequency of the impulses increased with increase of stimulus.

It was suggested by them that, if the sensation varied as the frequency of the impulses, a gradation in sensation would be thus provided. If this be true it is to be expected from Fechner's law as well as from other work that the frequency of the impulses as measured by them should vary as the logarithm of the stimulus.

[6] The impulses produced by Sensory Nerve endings Parts 1 and 4, Jour. of Physiol., *61*, No. 1 and *62*, No. 1; 1926.

[7] Same Title, Parts 2 and 3, Jour. of Physiol., *61*, No. 2 and 4; 1926.

Their data on the response of a single end organ and of few end organs, which are given here in Table 4, are plotted on a semi-logarithmic scale in Fig. 5. The letter naming each graph is the same as that at the top of

TABLE 4

From—The Impulses Produced by Sensory Nerve Endings, Part 2, Journal of Physiol., Vol. 61, No. 2, 1926.

	Exp. 12 p. 162	
Wt. gram	*A* Initial frequency (Maximal) per sec.	*B* Frequency after 10 secs. loading
5	(190)	(120)
5	150	104
.5	35	15
1	66	37.5
2	98	59

	Exp. 12 p. 163	
	C Initial frequency (Maximal) per sec.	*D* Frequency after 10 secs. loading
2	37	17
1	26	16
.5	14	8

	Exp. 1 p. 158, Single End Organ	
	E Frequency	
1	33	
.5	27	
.25	21	

the column of the corresponding data. All their data given in part 2 of the series of papers[7] are plotted here except that for experiment 10, page 164, in which the measurements were made on the muscle intact, and in which the number of end organs may have differed with the different loads.

It will be observed that the data conform very well to the relation,

$$f = k \log W.$$

where f is the frequency of the nerve impulses and W is the value of the weight used as stimulus. The graphs A, B and E which represent the relation immediately after the weight was applied are probably

more representative of normal body conditions than the other two. The points which lie above the graphs *A* and *B* may be due to the action of additional end organs, as the authors[7] considered it probable that the large stimuli brought additional end organs into action when they were present. The graph *E* known to be for a single end organ conforms exactly to the equation.

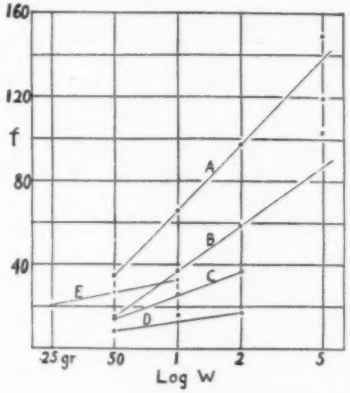

FIG. 5

The number of cases given is insufficient to be taken as very conclusive evidence, but it appears quite probable that the relation exists. It, therefore, seems likely that the elementary effect,

$$n = c \log I$$

of this discussion is due to the frequency of the nerve impulses from a single end organ or an associated set of end organs.

If this be true the magnitude of *S* is due to the summation of the effects due to the frequencies of the impulses along the particular nerve fibres in operation at any time. Also the effects due to the frequencies of the impulses are additive whether or not the end organs from which they arise have the same or different thresholds.

It is noteworthy in this regard that, if the Porter effect elements have been properly interpreted as being due to nerve impulse frequencies, it may be concluded that the frequency increases with stimulus in exactly the same way in all the conducting elements in action with any particular stimulus. This follows from the fact that the elements for each color were found to be identical. The evidence from the touch graphs would indicate that the unit of the tactile nerve system may react in the same way exactly, in all normal individuals.

The discovery of phenomena by Allen[8] which he has explained in terms of visual reflexes suggests a way of accounting for the negative elements,

$$n = -k \log I.$$

Allen[8] found that stimulation with light of the α intensity caused depression of the three primary sensations, while stimulation with the β and γ intensities produced their enhancement, also that the δ intensity caused enhancement of the colors being used as a stimulus and depression of the complementary, while the ϵ intensity produced the opposite effects. There were two exceptional colors 505 mμ and 570 mμ which he called equilibrium colors since stimulation by them caused neither enhancement nor depression.

If these effects are due to reflex action as Allen has concluded, and since it is to be expected, perhaps, that S will be modified by the diversion of energy to this purpose it seems not unlikely that the negative components of the Porter graph can be properly identified as reflexes. This seems particularly likely since the equilibrium color 505 mμ gives no reflex effects or they are in equilibrium while at the same time its graph consists of a single straight line as far as it has been determined. The graph for the equilibrium color 570 mμ has one new branch at very high intensity, but since the slope of the new branch is less than the other by an even number of elements (table 1) it may be that the reflexes are still in equilibrium.

This view that the negative elements are reflex actions is not incompatible with the idea that the gradation of the elements of S may be due to the change of frequency of the nerve impulses. This is so, either, if the effect of the impulses is divided in the cortex so that part gives rise to sensation and part gives rise to impulses along the efferent nerves, or, if a similar division occurs in the synapses if it be allowed that the impulses are not conducted across the synapses but rather that the synapses are affected in such a way that they in turn give rise to new impulses. For if a frequency-stimulus relation,

$$f = k \log W$$

exists along the nerve fibre up to the synapse and if an effect E occurs at the synapse which corresponds directly to f, then will

$$E = k \log W.$$

[8] Am. Jour. Physiol. Op., 5, No. 3.

If now the quantity E be dissipated in integral parts lE and mE due to its causing impulses along two paths and if the frequencies along these paths f_1 and f_2 correspond directly to lE and mE respectively, then will

$$f_1 = lk \log W.$$

and

$$f_2 = mk \log W.$$

And if f_1 gives rise to S while f_2 is diverted to some other purpose, f_2 will appear negative in the analysis of S and will thus correspond to the negative effects,

$$n = -c \log I$$

of this discussion.

It is significant in this regard that the negative and positive elements appear from the slope relations to be identical except for sign. This may be taken as evidence that if the one kind embody the law of the relation of nerve impulse frequencies with stimulus giving rise to the effect S, the other kind apply to the same kind of nerve action which is subtractive from S either directly or by diversion at some point along the afferent path.

Whether or not the negative elements exist only in the intensity ranges where the slope of the branch of the Porter graph is smaller than the previous one cannot be ascertained from the data. If they give rise to reflexes it is necessary that they should exist with all the intensity ranges since all intensities cause reflex effects.

The previous explanation of the existence of two branches in the Porter graph in which the one branch was considered to be representative of rod vision, while the other was due to cone stimulation, although attractive in the light of the duplicity theory, seems to be no longer adequate. This is especially true since it has been found by the critical frequency method that similar graphs consisting of two or more branches exist in taste, touch, hearing, and the pain and temperature senses. This perhaps cannot be considered as evidence against the view that the α branch in vision may be due to rod action and that the other branches are due to different sets of cones with different thresholds, but it makes it very doubtful.

The interpretation given here of the branches and of their slope relations possesses the following advantages: (1). It is conformable to the, "all or none," law. (2). It allows of a gradation of end effect even with a single conductor in action by assuming the effect to corre-

spond directly with impulse frequency. (3). It overcomes the inadequacy of nerve impulse frequency to give sufficient gradation of sensation by the addition of new actions from new end organs with different higher intensity thresholds, while at the same time there seems to be good evidence of the existence of these different thresholds. (4). It includes in its scope the action of efferent nerves which Allen's evidence of sensory reflex effects requires. (5). It is extremely simple in that, although they may start at different thresholds, all the elements making up to total effect are identically the same for each color at least and are perhaps the same for all colors.

If the interpretation is the true one it is apparent that the critical frequency method in vision and the similar methods devised by Allen to investigate the other sensoria may be used to obtain very accurate information by the analysis of the Porter effects on either the response of a single end organ or of an associated set of end organs, to whichever the elementary effect is due. These data on the action of the nerves in living bodies in normal condition should be very valuable in conjunction with those usually obtained by physiologists under abnormal conditions as they are necessarily much more accurate although usually more difficult to explain.

To those who accept the definition of sensation by the quantity S of this discussion it will be evident that the elements of S conform exactly to the simple form of Fechner's law,

$$S = K \log I$$

The deviations from Fechner's law are, from the ideas advanced here, due to no account having been taken of the entering of new additive effects at higher thresholds. The more complex form of the law required to include all the added effects through a large intensity range is still comparatively simple on account of the integral relation of the constants.

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ON BEAD SPECTRA

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ABSTRACT

By the use of double fluxes instead of single fluxes an attempt was made (a) to better the resolution and (b) to increase the intensity of the transformation spectra of several elements. These objects were partially attained, and the principle of essential identity, as applied to this type of spectrum, was substantiated.

The first part of our work consisted of a search for fluxes which are molten at temperatures between 700°C and 1300°C, and which possess the properties of solubility for the oxides and of optical transparency. Sodium fluoride, sodium silicate, calcium fluoride, or sodium carbonate were successively mixed with such substances as boric acid, borax, or potassium phosphate to form binary fluxes. Each bead was examined for the above mentioned properties and a chart of the beads was prepared for our further use. The second part of our work consisted of a study of the transformation spectra of approximately twenty elements when dissolved in these fluxes. The wave-lengths of bands which appeared in the spectra of the unactivated fluxes were first noted, that such bands might later be subtracted from the spectrum when the bead contained the dissolved activator. The method of mounting the beads and of observing the transformation spectra was similar to that employed by Nichols, Howes, and Wilber.[1] A tiny jet of hydrogen, burning freely in open air, without admixture of oxygen by jet, was used as a source of excitation.

The search for fluxes which would permit narrower bands when such favorable elements as erbium, neodymium, terbium, samarium, europium, and thulium were introduced proved to be a difficult one. The transformation spectra were formed at quite different temperatures; one might be initiated by a change in the structure of the flux, another by a change in the structure of the substance containing the activating element, etc. Two transformations often occurred during one cooling process. Except in the case of the rare earth elements the spectra were ontinuous, with one region of dominating intensity. The double fluxes proved to be more useful than the single fluxes because of their

[1] Nichols, Howes, and Wilber, "On Transformation Spectra," J.O.S.A. & R.S.I., *14* p. 205; 1927.

greater solvent action, and because of their longer periods of transformation. The double fluxes were also quite transparent when molten, and often even after solidification. It was found advantageous to make a flux of one component known to give transparency with another component of great solvent action, e.g., potassium phosphate with sodium fluoride. Such a bead was excellent for work at temperatures approximating 1000°C. Another similar flux consisted of potassium di-hydrogen phosphate with calcium fluoride, but the phosphate had to be replenished occasionally to prevent the oxidation of the fluoride.

The transformation spectra are not to be confused with the emission spectra of the incandescent oxides. When the latter type of spectrum appeared, as in the process of repeatedly warming and cooling a bead containing erbium oxide, the bands were observed and compared with those of the transformation spectrum of erbium. For the lowest temperatures at which the beads were studied we generally used a flux containing boric acid or borax; e.g., a flux of boric acid with one of the fluorides.

Erbium oxide was first introduced into a bead of calcium fluoride with sodium fluoride. A spectroscopic examination of the blank flux, i.e., the flux without the erbium, showed a dim continuous spectrum, with the red region predominant, but without bands. With erbium oxide 10 per cent the bead showed a continuous spectrum at low temperatures, but with warming a very broad band in the green region developed. With further warming bands developed at .537, .526, .553, and possibly .556μ. The red portion of the spectrum did not resolve until the first transformation which accompanied the cooling process occurred, when a broad band emerged from the continuous spectrum at $.620 \pm 10$. On continued heating the volatile components of one side of the bead evaporated, and the strong narrow emission bands of erbium oxide appeared. By careful manipulation it was found possible to bring the oxidized portion of the bead into the field of the spectrometer contiguously with the unoxidized portion. The narrow emission bands of the oxidized portion were apparently contiguous with the broader crests of the transformation bands at .620 and .553.

The flux of calcium fluoride with sodium fluoride was not useful in studying the transformations at lower temperatures because it clouded too readily, hence the erbium oxide was dissolved in a flux of boric acid with calcium fluoride. The transformation spectrum came into the field at a lower temperature. The bands were of approximately the same wave-length as those of the earlier specimen.

A bead of potassium di-hydrogen phosphate mixed with calcium fluoride presented a very faint continuous spectrum and colored the flame a delicate blue. The spectrum of this vapor was very feeble. The transformation bands emitted by erbium in this flux may be found in Table 1. After one had thoroughly dissolved the erbium oxide the broad red band appeared on cooling at $.620 \pm 10$. The band could be made to appear repeatedly on cooling and then could be "shifted" towards the longer wave-lengths with a slight increase in the heat. The term "shift" as applied in this paper means a relative change in the intensity of the underlying components of a given transformation band.

Erbium oxide, 5 per cent, was dissolved in a bead of sodium carbonate with potassium di-hydrogen phosphate. The spectrum of the blank bead was of the usual faint continuous type, with no enhanced bands on cooling or heating. The red band again formed at the higher temperatures and the green bands at the lower temperatures. While one of the observers watched the formation of bands in the field of the spectrometer the other observer looked for optical changes in the bead. Interestingly enough, these bands only came out during a transformation accompanying the clearing of the bead. Once the bands in the green had been formed it was found that an increase in intensity could be produced by a fall in temperature, but unfortunately the resolution was poorer.

A bead of potassium di-hydrogen phosphate and sodium fluoride gave a faint continuous spectrum. A slight amount of erbium oxide was added and at once the red band at .651 appeared. On quenching the flame the band "shifted" toward the shorter wave-lengths showing a marked similarity in behavior to that of the erbium band at .620 in potassium di-hydrogen phosphate with calcium fluoride. The resolution of the green bands was particularly good.

Neodymium was next studied. Neodymium oxide was added to a bead of di-basic sodium phosphate and warmed until completely dissolved. Bands appeared at $.655 \pm 4$, at .601 and at $.573 \pm 3$. On cooling the bead a band appeared at $.587 \pm 3$ after the sodium line had been removed from the field by dropping the flame. During the transformation a strong broad band formed at $.533 \pm 2$ and then died rather quickly. The resolution of the neodymium spectrum was nearly as decisive as that of erbium in the same flux. The spectrum becomes vague if the temperature is raised a little too high. More neodymium oxide, to approximately 10 per cent was added and although the bead

appeared more intense in color the spectrum appeared less resolved than when less activator was used. This is a particular attribute of luminescent radiation, as has often been shown by Urbain and others

TABLE 1. *Transformation Bands. Element: Erbium.*

Flux	Band 1	Band 2	Band 3	Band 4	Band 5	Band 6
CaF_2+NaF	—	.620	—	.553	.537	.526
$KH_2PO_4+CaF_2$	—	.620	.579	—	.533	.523
$KH_2PO_4+Na_2CO$	.650	.625	.578	.552	.532	.522
KH_2PO_4+NaF	.651	.630	.577	—	.534	.522
$H_3BO_3+Na_2CO_3$	.652	.623	.578	.550	.532	.521
Na_2H+PO_4	.651	.620	.575	.549	.533	.521

when a solid solution of a rare earth yields a spectrum under the excitation by cathode rays. The 10 per cent neodymium solution exhibited the same bands as the bead of weaker concentration, with the addition of a very narrow band at $.630 \pm 1$ observed only during melting.

Neodymium was next dissolved in mono-basic potassium phosphate with sodium fluoride. The resolution was better than in di-basic sodium phosphate beads. After a second heating the resolution became less definite and the bead became rather opaque. In Table 2 will be found the principle bands of neodymium in six fluxes.

TABLE 2. *Transformation Bands. Element: Neodymium.*

Flux	Band 1	Band 2	Band 3	Band 4	Band 5	Band 6	Band 7
Na_2HPO_4	.653	.630	—	.602	.587	.571	.533
KH_2PO_4+NaF	.651	—	.612	.603	—	.570	.533
$H_3BO_3+Na_2CO_3$	—	—	—	.600	—	.573	.535
H_3BO_3+NaF	.651	—	—	.602	.586	.572	.535
$NaH_2PO_4+K_2SO_4$	.655	—	—	.602	—	.574	.533
$KH_2PO_4+CaF_2$	.655	.631	.612	.604	.587	.575	—

A bead consisting of 5 per cent neodymium oxide in boric acid with sodium carbonate exhibited the type of "shift" mentioned with the erbium bands; i.e., the $.600 \pm 2$ band moved towards greater wavelengths with rise of temperature. The extreme red of the spectrum was not resolved, hence the effect, if present in that region, could not be detected. An increased amount of boric acid made the bead extremely clear but did not improve the resolution of the bands.

The spectrum was studied when emitted from a bead containing neodymium in a flux of sodium fluoride with boric acid at a slightly higher temperature. During a gentle heating of the bead the band at $.586 \pm 1$ appeared suddenly and then as suddenly disappeared. The enhancement of the band could be observed several times on alternately heating and cooling the same bead.

A flux consisting of di-hydrogen potassium phosphate with calcium fluoride proved to be best suited for our work with neodymium oxide. The bands were all narrower and occasionally doubled, e.g., the band listed in Table 2 as .631 was at one transformation viewed as a doublet with components at $.627 \pm 3$ and $.634 \pm 2$. This flux brought out a new band at $.485 \pm 4$.

Terbium was studied next. It immediately presented a spectrum which could not be resolved so well as the spectra of erbium and neodymium. An added difficulty was met when it was found that the transformation periods were shorter, and less easily controlled. In a flux consisting of borax with boric acid the transformation could be controlled fairly well, but the separation of the bands was not complete.

TABLE 3. *Transformation Bands. Element: Terbium.*

Flux	Band 1	Band 2	Band 3	Band 4	Band 5	Band 6
KH_2PO_4+NaF	——	——	——	.575	.558	.540
$Na_2B_4O_7+Na_2CO_3$	.628	——	——	.575	.555	.545
$NaF+CaF_2$	.624	——	.598	.572	——	.540
$KH_2PO_4+CaF_2$	.625	.611	.599	.570	.556	.537
$Na_2B_4O_7$	.626	.611	.602	.574	.553	.539

Table 3 gives the wave-lengths of the terbium bands in five fluxes. Because of the nature of the bands the measurements can not be considered as accurate as those of the bands of erbium or neodymium. The band at $.626 \pm 2$ was the only band which persisted.

Samarium oxide in sodium fluoride with boric acid exhibited four well defined bands at $.645 \pm 2$ and $.602 \pm 2$ and $.570 \pm 5$ and $.518 \pm 3$. The band of wave-length .570 was often doubled at .560 and .576. Two broad bands appeared at .530 and .480. The intensity of this spectrum was great enough to allow an extensive study of the bands. The wave-lengths will be found in Table 4. The very broad band at .530 often doubled to form bands No. 5 and No. 6. Such a type of doubling in these spectra was not generally obvious, yet it was our belief that the tendency was always present, which led us to the con-

clusion that the variations in the measurements on a given band are due in part to a real change in the relative intensities of two or more components. The control of the intensities of such component bands is, for the present, beyond possibility. In addition to the bands of samarium which are listed in Table 4 we measured a broad band at

TABLE 4. *Transformation Bands. Element: Samarium.*

Flux	Band 1	Band 2	Band 3	Band 4	Band 5	Band 6	Band 7
$NaF+H_3BO_3$	.645	—	.602	.576 .560	.546	.518	.480
$NaF+CaF_2$	.644	.615	.600	.569 .560	.546	.517	.477
$Na_2SiO_3+Na_2B_4O_7$	.645	.612	—	.573	.546	.517	.475
$NaF+K_2SO_4$	—	.612	—	—	.543	—	—
$NaF+K_2SO_4+Na_2SiO_3$	—	.610	.601	.570	—	—	—
$NaHPO_4$	.642	.612	.601	.575 .558	—	.518	.471
K_2SiO_3+NaF	—	—	—	.573 .560	—	.511	—
$Na_2SiO_3+K_2SiO_3+NaF$	—	—	—	.572	—	.511 .517	—

$.430 \pm 8$ in both mono-basic sodium phosphate and in a flux of sodium fluoride with potassium sulphate. Another broad band, located at $.452 \pm 5$ was found in the samarium spectrum from mono-basic sodium phosphate and from a flux of calcium fluoride with sodium fluoride.

TABLE 5. *Transformation Bands. Element: Europium.*

Flux	Band 1	Band 2	Band 3	Band 4	Band 5	Band 6	Band 7
KH_2PO_4+NaF	—	.611	.600	.587	.560	.530	—
CaF_2+NaF	.625	.611 .619	.600	.584	.558	.535	—
Na_2HPO_4	—	.612	.604	.587	—	.535	.472
$KH_2PO_4+CaF_2$	—	.613	—	.583	.560	.530	.474
$H_3BO_3+Na_2CO_3$	.630	.612 .617	.602	—	—	—	.473

The spectra of europium oxide in five different fluxes will be found in Table 5. Europium beads often showed beautiful coloring in the blue and green, but the resolution was less favorable than that of the erbium and neodymium. A flux of calcium fluoride with sodium

fluoride gave the europium spectrum with a fair degree of definiteness at the transformation points but the bead evaporated too rapidly for our observations to be completed. For this reason we were forced to choose the less desirable method of reading a part of the spectrum, then renewing the bead in the same proportions and continuing the measurements in another part of the spectrum. Approximately forty specimens of europium in this flux were studied. It may be noted that Band No. 2 at .613 was often read as a doublet with components at .611 and .619. This band was also doubled in a flux of boric acid with sodium carbonate, but was very transient. The other bands were set as if of one component only.

Thulium in borax yielded a beautiful red bead. Two broad bands, at $.680 \pm 10$ and $.620 \pm 9$ respectively, were first measured. Other bands at $.519 \pm 1$ and $.572 \pm 2$ and $.472 \pm 2$ were readily measured. The band at .437 was very broad and hence difficult to measure. These bands are listed in the uppermost line in Table 6. The bands of thulium were not so difficult to measure as those of terbium, but still lacked the decisiveness of those listed in Tables 1 and 2. The band at .519, Table 6, was measured as a doublet in three different fluxes. This doublet was of

TABLE 6. *Transformation Bands. Element: Thulium.*

Flux	Band 1	Band 2	Band 3	Band 4	Band 5	Band 6	Band 7
$Na_2B_4O_7$	.679	.620	.600	.572	.519	.472	.438
KH_2PO_4+NaF	——	.625	.600	.575	.520	.473	——
CaF_2+NaF	.670	.628	——	.568	.525 .512	.470	——
$KH_2PO_4+CaF_2$	.682	.627	.604	.575	.525 .514	.477	.437 .448
$Na_2HPO_4+KH_2PO_4$	.683	.627	.603	.570	.524 .512	.475	——

an unusual nature, since one component appeared at a different temperature than the other component. The component at $.514 \pm 2$ was rather diffuse and transient. The component at .525 while also rather broad was somewhat better defined and more persistent.

CONCLUSIONS

1. The positions of the transformation bands of an element are approximately the same in all fluxes, as may be observed from each of the tables, hence the principle of "essential identity" applies.

2. Variations in the wave-length of a given band may be due to real variations in the relative intensity of two hidden components. In some spectra it was found possible to separate the components.

3. Conditions in a bead favoring perfect solubility for a rare earth and complete transparency are not sufficient to yield such spectra: there must also be a transformation.

4. The use of the double fluxes has enabled the authors to better control and retard the transformations.

The specimens of the rare earths used in this study were isolated in the laboratories of Professor Charles James of the University of New Hampshire. The authors express their thanks to Professor E. L. Nichols for suggesting this beautiful yet delicate problem.

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ITHACA, NEW YORK.

Problems of Modern Physics. By H. A. Lorentz. (Edited by H. Bateman.) vi+312 pages. Ginn and Company, Boston. $3.60.

This volume is based upon a series of lectures delivered by Dr. Lorentz at the California Institute of Technology in 1922. The text of the lectures —some 200 pages—is supplemented by nearly 100 pages of appendices, prepared since the lectures were delivered.

The title of the book is its best *brief* description. Among the topics discussed in the sixty-four sections may be mentioned (almost taken at random) The Electromagnetic Equations, Flow of Energy in a Beam of Light, The Production of Light, Reaction of Radiation on a Moving Electron, Scattering of Light by Molecules of a Gas, The Relativity Transformations, Structure of the Electron, The Radiation of Angular Momentum, New Assumptions Concerning the Motion of Light Quanta, Theory of Dispersion and Absorption, the Zeeman Effect, etc. The treatment, while of course largely mathematical, is given in the delightfully clear style so characteristic of the author.

Although much of the material was prepared five years ago, the careful student of modern physics will find the book a gold mine of information which, as the author hopes, will serve "as an introduction to the problems on which the efforts of physicists are now largely concentrated."

F. K. RICHTMYER

INSTRUMENT SECTION

"UNIVERSAL" FREQUENCY STANDARDIZATION FROM A SINGLE FREQUENCY STANDARD†

By J. K. Clapp

ABSTRACT

In this paper a method is described of obtaining from a single frequency standard source (quartz plate oscillator) a very large series of frequencies related to the known frequency by a ratio of simple integers. The frequencies thus attainable range from a few hundred to several millions of cycles per second, covering the entire gamut of audio and radio frequencies now in regular use. Not only does the method provide a means for the calibration of oscillators in this range of frequency, the calibration of wavemeters (or frequency meters) and means for obtaining a desired frequency, or frequencies, for laboratory measurements, but also a means of effecting a direct comparison of frequency standards. The most important result is that such frequency standards may be compared even though they are of widely different frequencies, as for example, it makes possible the comparison of a standard of audio frequency with a standard of radio frequency. The application of the method in laboratory measurements makes it possible to refer all work in which determinations of frequency, or steps of frequency are required directly to a single standard of frequency. *Simultaneous* comparison of an audio frequency standard, a radio frequency standard, and standing waves on parallel wires is possible.

I. PROPERTIES AND APPLICATIONS OF HARMONIC SERIES

In considering the method to be used in obtaining a large number of frequencies from a single known frequency, it may be well to set forth some of the properties of the harmonic series used to represent the current wave of a distorted wave generator. Such a distorted wave may be represented, by Fourier's analysis, by a series of terms involving a fundamental frequency and a series of harmonic frequencies which are integral multiples of the fundamental frequency.

If the fundamental frequency of the generator is f, then the frequency of the nth harmonic is nf, and in general terms, the frequency of the next higher harmonic is $(n+1)f$. The difference in frequency between any two successive harmonics is then

$$[(n+1)-n]f=f$$

that is, equal to the fundamental frequency of the generator. This fact is of considerable practical importance in approximately determining the unknown fundamental frequency of a generator, since if the frequencies of a series of the harmonic frequencies are determined,

† Contribution from the Col. E. H. R. Green Shortwave Research.

the successive differences represent approximate values of the fundamental frequency. Having this approximate value, the observed frequencies may be divided by this fundamental frequency and the harmonic order numbers obtained. These order numbers must be integral, so the appropriate integral value is designated for the experimental value (which may not be integral), for each of the observed frequencies. Dividing the observed frequencies by these order numbers results in a series of values for the fundamental frequency, which will be much nearer the true fundamental frequency. As a final step the average of these fundamental frequencies may be used and the result will be very near the true fundamental frequency, even though a comparatively few observed frequencies were obtained.

If a series of harmonic frequencies is to be used in calibration work, or in any other work where a series of steps of frequency is desired, it is of interest to note the percentage change in frequency which occurs in passing from one harmonic to the next higher. If we pass from the nth harmonic to the $(n+1)$st, the change in frequency is f (the fundamental frequency), and referred to the first harmonic frequency nf, this change is, in percent:

$$(f/nf)100 = 100/n \qquad (1)$$

For much of the work of calibration of frequency devices, it is essential that the change in frequency taken as a step in the calibration be not too large or too small. By expression (1) it is seen that for low values of n the percentage change in frequency is very large, while for large values of n it becomes very small. For certain purposes, then, such a series of harmonics will give suitable steps of frequency only over a limited range of frequency. In calibrating wavemeters, for example, the steps in frequency should not be made too small, first because of the inordinate labor of covering any appreciable range of frequency, and second because of the ultimate resolving power of the instrument, which ranges from 0.1 to 1 percent. Most of the work of this nature may be carried out with minimum changes of frequency of not less than 1 percent, before these limitations become effective.

For purposes of increasing the resolving power of a tuned circuit with respect to frequency, the circuit may be used in a regenerative vacuum tube assembly. In this manner harmonics of order higher than the 100th may be strongly amplified and used for "stepping" over greater ranges of frequency, from a given fundamental frequency, than would otherwise be possible.

To obtain a great range in frequency some modification of the harmonic series of a single frequency standard source must be made. Changing the fundamental frequency of the source, to various known values, results in separate series of harmonic frequencies for each value of fundamental frequency, and by proper choice of the fundamentals, a great range of frequency may be covered. The objections to this method, for accurate work, lie in the difficulty of obtaining and maintaining the precision of a variable frequency standard source, or of separate standards of various fundamental frequencies. The process of obtaining a desired range of frequency by this method may be described by the series of harmonics associated with the various known fundamental frequencies, f_1, f_2, etc., thus:

$$n(f_1),\ n(f_2), \tag{2}$$

where n is the harmonic order number in each series.

If, instead of attempting to obtain the desired range of frequency by adjustment of the fundamental frequency to the values $f_1, f_2, \ldots$. etc., we modify the factor n so that the harmonic series of a single frequency standard source becomes

$$f_n = n(f/m) \tag{3}$$

where both n and m are integers, we have a means of obtaining a very large number of frequencies which are not integral multiples of the fundamental frequency f, but which are nevertheless related to that frequency by a ratio of simple integers.

The effect expressed by (3) is obtained many times in practice, when, for example, an auxilliary heterodyne oscillator is coupled to the source. Listening in telephone receivers placed in the output of the heterodyne oscillator, as the frequency of the heterodyne is varied, a series of zero beat points is obtained on the dial of the heterodyne, where

$$nf = m_h f_h \tag{4}$$

where f is the fundamental of the source;

n, the harmonic order number of the frequency referred to the source fundamental;

f_h, the fundamental frequency of the heterodyne;

and m_h, the harmonic order number of the frequency referred to the heterodyne fundamental.

The important point to notice is that the two members of the expression (4) refer to physically separated circuits, the left hand member being

associated with the source, while the right hand is entirely associated with the heterodyne.

Interpreted in another way, it may be said that the fundamental frequency of the heterodyne f_h will be related to the source fundamental by

$$f_h = n(f/m_h) \tag{5}$$

only so long as the heterodyne remains coupled to the standard source and maintained at that adjustment which satisfies this relationship. By permanently associating the heterodyne oscillator with the standard source, it would be quite simple to obtain this adjustment when desired. Even then, in making use of this frequency f_h, to refer it to some other frequency, or to calibrate a wavemeter, reaction of the measuring circuit on the heterodyne would cause the frequency to break away from the value giving zero beat, and the condition (5) would not hold. It would be necessary to observe the beat in the telephones, or by other means, and make certain that it did not depart from zero if the exact relationship is not to be disturbed.

For high frequency manipulation the degree of precision represented by "zero beat" as determined by aural methods is more than sufficient for usual purposes. The frequencies are usually within ± 50 cycles, taking account of the characteristics of the ear and telephones, when no "tying-in" or synchronization of the oscillators takes place. For more precise determinations of the "zero beat" condition use should be made of a detector and galvanometer for visual indication of the low frequency beats.[2]

An alternative procedure for obtaining an indication of very low frequency beats by means of telephone receivers is to make use of a third heterodyning oscillator.[26] This oscillator is arranged to be subjected to voltages from both the standard oscillator and the beating oscillator, producing beats with each. If the beating oscillator is not exactly at the frequency of the standard, a slow waxing and waning of the beat tone heard in the third oscillator takes place, since the two beat frequencies are very nearly identical. Adjustment is made until the period of the waxing and waning is reduced to zero, or to as low a value as is necessary for the accuracy desired.

If the standard frequency source *could be made to control the frequency* of the heterodyne oscillator, then the heterodyne oscillator would become an essential and integral part of the standard frequency source and expression (5) would then be properly written as (3), that is,

$$f_n = n(f/m)$$

where all of the quantities *refer to the standard source.*[15,16,17] In carrying out this process, it is not convenient to make use of the usual type of vacuum tube oscillator, such as would be used for the heterodyne, since such an oscillator is not readily influenced by the introduction of a small harmonic voltage into the circuit. Furthermore, such an oscillator has a pronounced fundamental frequency, with relatively small amplitude harmonics.[18] By making use of an "oscillator" which has no highly resonant circuit, it is found possible to obtain the desired control more easily and over greater ranges of adjustment. Such an oscillator is the "multivibrator" described by Abraham and Bloch.[16] This device is simply a two stage resistance-coupled amplifier having the output connected directly back to the input. As the current in the plate or grid circuits is very far from sinusoidal, the harmonic amplitudes are relatively large (of the order of $1/n$) and as the circuit contains no appreciable inductance the frequency generated is greatly influenced by any factors which change the effective resistance or capacitance of the circuit, such as heating in the tubes and resistors, variations in supply voltages, and so on. This instability of frequency, due largely to the absence of any inherent resonance, has curtailed the application of this device in many instances where precise results were desired.

A simple source rich in harmonics is obtained by forcing the output of an oscillator through a rectifier, as has been done by Chaffee and Breit.[24,4] The multivibrator has a certain advantage in that the relative harmonic amplitudes are approximately given by $1/n$ for both odd and even harmonics, where the rectifier output tends to emphasize the odd harmonics. However, the most important property of the multivibrator is that it may be so readily controlled by the frequency standard.

The very instability which formerly restricted the use of the device is now highly desirable, since the multivibrator may easily be forced to operate on a fixed frequency by the introduction of a small harmonic voltage from a fixed frequency source. The frequency of this controlling voltage need not be that of the fundamental frequency of the multivibrator but may be that of any harmonic of the multivibrator up to the 40th or higher. Previous applications of the controlled multivibrator have been made in standardizing radio frequency devices from a known audio frequency source.[15,17] For example, a tube driven tuning fork is utilized to control the fundamental frequency of a multivibrator; the 20th harmonic of this multivibrator is used to control the fundamental of a second multivibrator, the harmonics of which are then 20

times as far apart on the frequency scale as the harmonics of the first, or twenty times the reference audio frequency. With a 1 kc fork, these would be 20 kc apart and could be used directly to compare frequencies up to the order of 2,000 kc, (or 150 meters).

By choosing the reference frequency many times higher, of the order of 100 kc, and making use of a quartz plate oscillator, both the range and the constancy of the resulting frequencies are greatly increased. The range is greatly increased since we may step down from the frequency of the standard toward the audio frequencies, obtaining frequencies in the lower range of audible frequencies, or we may step up from the standard frequency toward the very high radio frequencies, arriving at frequencies which may be determined from standing waves on parallel wires set up within the laboratory.[9,10,22] The frequency of the quartz plate oscillator is practically independent of adjustments of the tube supply voltages; the variations with temperature are remarkably small,—of the order of a few parts in a million per degree Centigrade for a suitable cut of the crystal. Starting from a frequency which is roughly in the middle of the harmonic spectrum, the lower frequencies are attained with an absolute variation in frequency which is much less than that of the standard frequency (though the percentage variation is of course the same as that of the standard). The higher frequencies are reached with absolute variations in frequency equal to that of the standard, multiplied by the harmonic order used (though again the percentage variation is that of the standard). The very high frequencies are reached from a high frequency standard with smaller variations than is possible from a low frequency standard, such as a tuning fork. By employing the high frequency standard we may reach audio frequencies by a frequency division of the same order as the order of multiplication which has been used by other investigators; extending the range to higher frequencies by the use of the same order of multiplication, gives an upper limit of frequency n^2 times as great as results from the use of frequency multiplication from an audio frequency source, utilizing the same order n in each case.

II. DESCRIPTION OF APPARATUS

Crystal Oscillators. Fig. 1 indicates three simple arrangements of crystal oscillator circuits.[1,3,12,13,23] Of the three, that of *A* is generally preferable as most reliable and giving the greatest output. In each case

the quartz plate, as mounted between two flat electrodes, is represented at Q.*

Reference to Fig. 6, Curve of dc plate current vs C, it is seen that the current rises, as C is increased, to a maximum value and then decreases sharply as the resonance position of C is approached. The current of fundamental frequency, in the L, C circuit increases with

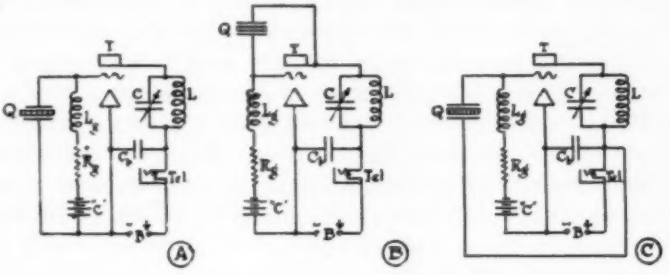

FIG. 1. *Circuits for quartz crystal oscillators.*

increased C, as shown by Curve I_1, reaching a maximum just before the resonance position of C is reached. When the resonance position is attained, the crystal stops oscillating, I_1 drops to zero, while the dc plate current resumes its steady value. A stable adjustment, with nearly the maximum output is obtainable with C slightly below the resonance value.

Standard Conditions for Crystal; Reactions and Ageing. For many purposes sufficient accuracy is attained without special precautions in operating the crystal oscillator.[13] Depending upon the manner in which the quartz plate is cut from the original crystal, various tem-

* The grid circuit of the tube includes a grid circuit choke coil, $L-g$; grid leak resistance $R-g$; and "C," or grid bias, battery C. Depending somewhat upon the type of vacuum tube used, T, either the resistance or the "C" battery may be dispensed with. If the tube is a small radio receiving tube, such as the UX-199 or UX-201A, with plate voltages of about 90, 15,000 or 20,000 ohms may be employed without battery. If a larger output is desired, a UX-210 tube, with about 200 volts plate supply may be used, in which case a few volts of "C" battery is advantageous, but not absolutely required. The plate circuit includes the tuned circuit L, C and the by-pass condenser $C-b$, in the ac portion. The tuned circuit must be capable of being adjusted to the fundamental frequency of the crystal, the greatest output being obtained in varying C from low toward high values, just before the resonance position is reached. The by-pass should be 0.01 mf or higher. The dc circuit includes the plate supply battery "B" and the telephone receivers (or a jack for plugging them into the circuit). In most cases it is not essential that telephone receivers be used at this position. The provision of a jack allows a dc milliammeter to be inserted in the plate circuit, the indications of which provide an easy means of determining when the crystal is oscillating.

perature coefficients of frequency will be obtained, ranging from practically zero to possibly one or two tenths of one percent per degree centigrade. In adjusting tuned circuits in the crystal oscillator, it will be found that the frequency will change slightly; once the adjustments have been made the frequency will remain constant. The frequency of the quartz plate is practically independent of the supply voltage variations, for ranges through the normal operating region of the tube employed. Since, from the method of making use of a single frequency of the crystal, but one standard need be maintained, it is not difficult, even in a small laboratory, to arrange the crystal in an evacuated container, and to maintain it at constant temperature. A glass container, with long arms containing the leads to the crystal, may be immersed in an ice bath, (or oil bath with thermostat control) so that reproducible conditions may be easily realized. Comparison

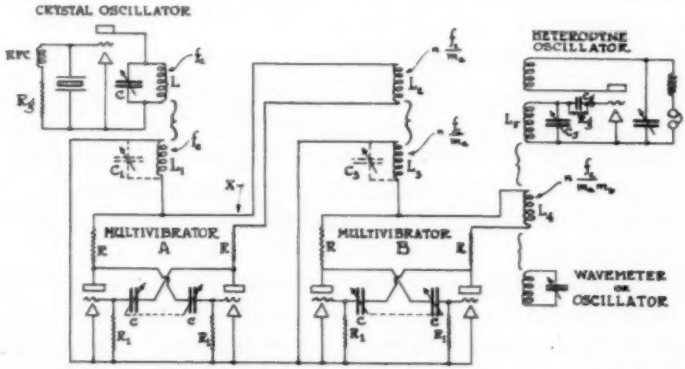

FIG. 2. *Arrangement of circuits for calibration purposes, for frequencies less than 100 f_c.*

of the standard crystal against an auxilliary crystal will yield the temperature-frequency curve of the standard; once this has been determined, it is not necessary to make use of the bath each time the standard is used. The adjustments of the tuned circuits should be provided with locking devices so that they may be fixed in position.

Reaction of the tuning circuit on the crystal, though small, may be avoided by the use of a screened-grid tube, or a neutralized tube of usual type, placed between the crystal tube and the tuned circuit from which the output is taken, as described by Hull.[19]

Variations of frequency of the standard crystal with ageing do not seem to be of sufficient magnitude to cause concern, even for precise

work, as indicated by Crossley in describing the work of the Naval Research Laboratory on this phase of the problem.[23,4]

Multivibrators. Fig. 2 indicates the circuits of the multivibrator,[16] as well as the arrangement of the circuit for obtaining audio frequencies from the crystal oscillator by means of two multivibrators in cascade.[15,17] Referring to the portion of the diagram surrounding the words "Multivibrator *A*," it is seen that the multivibrator consists of two vacuum tubes, each containing a resistance R in its plate circuit. The grid circuits contain resistances R_1 and condensers C, the grid circuit of one tube being connected to the plate of the other tube. In the common supply lead of the plate circuits is inserted a pick up circuit L_1, C_1, which is used to inject a small harmonic voltage of crystal frequency into the plate circuits of the tubes. In series with one of the plate circuits, as at X, a small coil is placed for obtaining the output from the multivibrator. In the diagram, the crystal oscillator, with the multivibrator circuit extending to the coil L_2, represents the arrangement and necessary equipment when a single multivibrator is used. It is convenient to have the condensers C variable through a common control, but as long as they are kept at approximately equal values, satisfactory operation will be obtained.*

Heterodyne Oscillators. A heterodyne oscillator of wide range of frequency is a practical necessity in all frequency calibration work. Such an oscillator is particularly useful when it is so arranged that the calibration will hold with fair accuracy,[18] over long periods of time and when the device has been calibrated in terms of harmonic order numbers of the standard, as well as in frequency. The circuit indicated in Fig. 2, of the tuned grid type, is convenient where plug-in coils are used for covering large ranges of frequency. However, it is not possible to keep an accurate calibration of such an oscillator, since the frequency varies with the degree of coupling between the coils and with the setting of the variable by-pass condenser.

Another arrangement, for obtaining an oscillator which will hold its calibration reasonably well, is indicated in Fig. 7. It is a Hartley circuit, in which large values of capacity are used, in order that variations of effective tube capacitance C_{gp}, will not greatly affect the frequency. The capacitance C_0 is of the order of 0.00035 mf, while the variable condenser C has a range of approximately 0.0015 mf.

* With UX-201A tubes, C maximum of 0.15 mf, supply voltage of about 90, Table 1 indicates the order of fundamental frequency in kilocycles per second obtained from the multivibrator:

Under these conditions, the frequency range with a given coil will be sensibly 2 to 1. The coils are arranged with three terminals, the ends and the center point of the winding being brought out to plugs, so that coils may be interchanged in the circuit.

TABLE 1. *Approximate fundamental frequencies (KC/sec) obtainable from a multivibrator using various resistences and capacitances. R = plate resistance, R_1 = grid resistance.*

C mf	*R*100000 $R_1$100000	*R*25000 $R_1$100000	*R*25000 $R_1$50000	*R*25000 $R_1$25000	*R*10000 $R_1$25000	*R*10000 $R_1$10000	*R*7500 $R_1$10000
.000125		30	48	70	107	160	220
.00025	15	16	33	58	55	96	125
.0005	5	10	18	30	34	60	80
.00075	2.5	7	12	18	23	35	57
.001		5	8	16	18	30	35
C mf	*R*100000 $R_1$100000	*R*50000 $R_1$50000	*R*5000 $R_1$10000	*R*2500 $R_1$5000	*R*2500 $R_1$5000	*R*2500 $R_1$5000	*R*50000 $R_1$50000
0.15	.06						
0.01		.9					
0.0004			140				
0.0003				360			
0.000140					570		
0.000095						735	
0.00038							15.4

It is usually more convenient to record the calibrations of the heterodyne oscillator in terms of the harmonic numbers referred to the standard source, than in terms of the absolute value of frequency. As indicated in Fig. 8, the calibration ordinates are ratios of n/m, where m had the value 4, and n ranged from 11 to 16. If the ratios are

expressed in lowest terms, the ratios which become integers indicate that the associated frequency is a harmonic of the crystal fundamental. Ratios which are not integral indicate "derived" harmonics. As the crystal harmonic frequencies occur throughout the scale of derived frequencies, these points serve as key points. Having these points identified by means of the heterodyne calibration, it is only necessary to count the zero beat points lying between any two integral numbers on the order number scale to obtain the value of m, which is equal to the number of these points plus one. The ratio n/m is thus completely determined very easily and quickly at any portion of the frequency scale covered by the heterodyne.

III. EXPERIMENTAL PROCEDURE

Determination of crystal fundamental (approximate). Suppose the crystal frequency to be of the order of 150 kc, the circuit arrangement of Fig. 2 for a single multivibrator and an auxiliary heterodyne oscillator available. We first take the case where the heterodyne oscillator

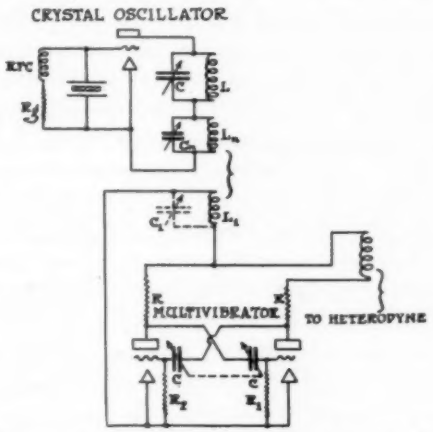

FIG. 3. *Arrangement of control circuit for frequencies greater than 100 f_c.*

has not been previously calibrated. Couple the heterodyne loosely with the crystal oscillator, operating the heterodyne at a few hundred kc, the multivibrator not being in operation. Record the condenser settings for the zero beat points (if loud and weak points are heard, record the loud ones as principals, and the weak ones as secondaries). With a wave-meter, or by any convenient means, determine the frequency associated with each of the principal zero beat points. By the

process outlined above (sec. I) determine the crystal fundamental approximately (if this is not known exactly), and thence determine the order numbers associated with each of the principal zero beat points. If the crystal frequency is known obtain the order numbers at once by dividing the observed frequencies by the crystal frequency.

Determination of synchronization of multivibrator with crystal. The heterodyne is now coupled to the multivibrator output coil and the multivibrator is put into operation. (If telephones are available in the plate circuit of the crystal oscillator, listen here while the frequency of the multivibrator fundamental is varied by adjustment of C.) Beat tones will be heard when the multivibrator does not synchronize with the crystal. These tones may be brought down toward zero beat but will suddenly disappear long before zero beat is reached, because the multivibrator falls into step with the crystal. Varying the frequency of the heterodyne results in two types of phenomena: First, a number of zero beat points may be found, where the characteristic tones of the beats are clear and steady,—having all of the steadiness of the crystal; second, a series of steady tones will be found, interspersed with tones which have a peculiarly characteristic roughness and unsteadiness. In the first case the multivibrator has synchronized with the crystal oscillator. Referring to the record of heterodyne settings for the crystal harmonics, adjust the heterodyne to pick up one of these. Pass to the next of the recorded points, counting the number of zero beat points encountered on the way. As described previously, this number is $(m-1)$. Increasing the number observed by unity gives the value of m for the particular multivibrator adjustment. In the second case, vary the multivibrator frequency until the series of clear steady beat tones is obtained, after which the above counting process may be carried out. Another method is given by Mallet, Andrews, and Rapson.[21]

If the value of m is suitable for the work in hand, the process of obtaining the desired series of frequencies is completed; if not, increase or decrease the multivibrator frequency until the desired value of m is obtained. Using a UX–201A tube for the crystal oscillator, and UX–201A tubes for the multivibrator, no difficulty should be encountered in obtaining all values of m lying between 1 and approximately 40.*

* Example of determination of m.

It may be that experimental data on the process of obtaining a suitable value of m for various purposes will be of value in making the method clearer.

This is equivalent to adjusting the multivibrator to approximately 40 different fundamental frequencies, all of which are definitely related

	Heterodyne oscillator settings for crystal harmonics	Heterodyne oscillator settings for multivibrator harmonics	Count of new zero beat points between crystal harmonics $=(m-1)$
n	98		
$n+1$	86	86	0
		75	1
		62	2 $m=3$
$n+2$	49	49	0
$n+3$	35		

Taking the adjacent crystal harmonic settings 86 and 49 of the first series, it is seen that in the second series two new points are found between these, giving $(m-1)=2$. m is then 3 and the fundamental frequency of the multivibrator is $(f/3)$, where f is the crystal fundamental frequency. The frequency of the nth multivibrator harmonic is then $f_n=n(f/3)$, where n is the order number of the harmonic in question.

The following examples serve to indicate the choice of the value of m for specific examples of wavemeter calibration. The choice of m regulates the interval between the successive harmonics on the frequency scale; if small steps of frequency are desired, for close calibration, then m should be fairly large; if only a few points are required in the given frequency interval, m should be relatively small.

Example of calibration of 40–100 kc Wavemeter. Suppose a wavemeter having a frequency range of approximately 40 to 100 kc is to be calibrated from a standard crystal of 138.5 kc. In the first place we note that the entire range of the wavemeter is below the fundamental frequency of the crystal, so that no direct comparison would be possible. Suppose, further, that frequency intervals of approximately 10 kc are desired, so that at least 6 calibration points will be determined on the scale of the wavemeter. Choosing $m=13$ or $m=14$, intervals of 10.65 and 9.89 kc result. If the value 13 is used, the fundamental frequency of the multivibrator is $138.5/13=10.65$ kc and the following harmonics yield frequencies in the desired region;

n	f(kc)
3	31.9
4	42.6
5	53.3
6	63.9
7	74.6
8	85.2
9	95.9
10	106.5

Example of calibration of wavemeter in broadcast band. As a further example, suppose a wavemeter or heterodyne oscillator is to be calibrated over the broadcast band of frequencies, i.e., from approximately 500 to 1500 kc, and that the desired frequency interval is about 50 kc. Again using the 138.5 kc crystal as the standard, we see that some of the crystal harmonics will lie in the desired range:

to the crystal fundamental frequency. If harmonics not higher than the 100th are used, then there are 4000 frequencies available from the crystal, though not all of them are unique. Even with this restriction

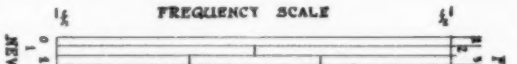

FIG. 4.

The positions of some of the possible frequencies, which are obtainable by the method described herein, lying between one harmonic of the standard, f_1, and the next higher, f_2, are shown diagramatically above. The factor m by which the standard frequency is divided in each case is given at the top of the columns, while the number of "new" or prime frequencies resulting is given at the bottom of the column. In the diagram, for any given value of m, those frequencies which are obtainable with some preceding value of m are indicated by dotted lines; in each case the prime frequencies are given by the full lines. Utilizing values of m up to 21, the number of prime frequencies obtained is 139; that is, in the frequency interval between successive harmonics of the standard, it is possible to intersperse, and utilize 139 other frequencies, all of which are known in terms of the standard of frequency.

n	f(kc)
4	552
5	693
6	831
7	969
8	1109
9	1246
10	1385

By choosing a value of $m=3$, the frequency interval is fixed at $138.5/3=45.15$kc, which is approximately the desired interval, and the multivibrator then yields harmonics in the desired range as follows:

n	f(kc)	n	f(kc)	n	f(kc)
10	461	18	831*	26	1200
11	508	19	877	27	1246*
12	552*	20	923	28	1293
13	600	21	969*	29	1339
14	646	22	1016	30	1385*
15	693*	23	1062	31	1421
16	739	24	1109*	32	1477
17	785	25	1153	33	1524*

where the points marked * may be obtained directly from the crystal, as indicated by the preceding table.

there are many more frequencies than would be obtained from the use of the three principal crystal frequencies[1,12] and harmonics of each up to the 100th (300) and the confusion resulting from the use of *three* principal fundamental frequencies is avoided. The position of the "derived" frequencies on the frequency scale, relative to two successive crystal harmonics f_1 and f_2, is indicated in Fig. 4.

Example of comparing a single frequency with crystal. Use may also be made of the possibility of choosing the ratio n/m to obtain a frequency very near to a desired frequency. Suppose it is desired to compare a crystal whose approximate frequency is 251.7 kc with the standard crystal previously used, 138.5 kc. If it is attempted to beat these crystals together directly, it will be found that audible beats will be obtained between them only when very high harmonics are used,—the 38th of the high frequency and the 69th of the low frequency crystal. While such a comparison is possible, it is not generally at all convenient, since equipment must be set up for frequencies far outside the region of the crystal fundamental frequencies.

If an audio frequency beat of 5000 cps or less is desired, it is only necessary to adjust the multivibrator fundamental to 10 000 cps (twice the desired beat frequency), for then the desired frequency can never be more than 5000 cps different from that of one of the multivibrator harmonics. While this process is sufficient to guarantee the production of a beat frequency of less than 5000 cps, it may not be necessary, or may not be the most convenient. If the ratio n/m may be given a suitable value, the desired beat may be obtained with a much greater frequency interval than 10 000 cps.

Following the first procedure, if the value 13 is given to m, the multivibrator fundamental becomes $138.5/13 = 10.65$ kc, $= 10650$ cps; the 251.7 kc crystal fundamental will then not be more than $10650/2 = 5325$ cps from one of the multivibrator harmonics.

However, by choosing $n/m = 20/11$ the multivibrator fundamental is 12595 cps, and the 20th harmonic gives 251908 cps which is very close to the desired value of 251700 cps. It is to be noticed that while the ratio 20/11 is close to that of 69/38, the ratio is not identical. Furthermore, the latter depends on *both of the crystals*, while the former depends *only on the standard crystal.*

Example of use of frequency ratios. In many types of laboratory measurements, where frequency ratios are involved, it is very convenient indeed to refer frequencies to a single standard of frequency, as in many cases the absolute value of the reference frequency need

only be known with sufficient precision to determine the harmonic order numbers, and need remain constant only during the period of the measurements.

For example, in determining the approximate apparent self-capacitance of a coil when viewed externally, by observing the resonance frequency of the coil when tuned with a calibrated variable condenser, (and extrapolating a plot of wavelength squared vs. added capacity) any order of harmonic may be used and the absolute value of the reference frequency is not needed provided the harmonic order numbers are known.

If the above process be carried out by computation instead of graphically, let the coil in question be tuned to a frequency f_1 by means of the variable condenser, where

$$f_1 = n\left(\frac{f_c}{m}\right) \text{ when } C = C_1$$

and then to frequency f_2, the next higher harmonic,

$$f_2 = (n+1)\left(\frac{f_c}{m}\right) \text{ when } C = C_2$$

where f_c = standard source fundamental, and n and m are integers.

Then,
$$f_1 = \frac{1}{2\pi\sqrt{L(C_0+C_1)}} = n\left(\frac{f_c}{m}\right)$$

$$f_2 = \frac{1}{2\pi\sqrt{L(C_0+C_2)}} = (n+1)\left(\frac{f_c}{m}\right)$$

and
$$\frac{f_1}{f_2} = \sqrt{\frac{C_0+C_2}{C_0+C_1}} = \frac{n}{n+1}$$

from which

$$C_0 = \frac{n^2C_1-(n+1)^2C_2}{2(n+1)} \text{ (in units of } C_1, C_2) \qquad (6)$$

If $n=1$ (fundamental of multivibrator) and $n+1=2$ (2nd Harmonic)[27] (6) reduces to

$$C_0 = \frac{C_1-4C_2}{3} \qquad (6A)$$

If high order harmonics are used, it is seen from expression (6) that the change in frequency in passing from one harmonic to the next is so small that the accuracy of the method is lost. If a greater ratio of f_2/f_1 is used, as by passing from the nth harmonic to the $(n+k)$th, the accuracy may be retained, without the necessity of a particular choice of the ratio n/m or of the harmonic order numbers n and $(n+k)$, thus:

$$C_0 = \frac{n^2C_1-(n+k)^2C_2}{k(2n+k)} \text{ (in units of } C_1,C_2) \qquad (7)$$

IV. RANGE COVERED BY CRYSTAL AND ONE MULTIVIBRATOR

Having considered the various adjustments for obtaining a series of frequencies in a desired range, or for obtaining a desired single frequency, it is well to consider the extremes of frequency over which such adjustments are possible. With a given crystal of fundamental f we may obtain a large number of frequencies lying between $f/40$ (approximately) and 100 f, if we limit outselves to a minimum separation of 1.0% between successive high order harmonic frequencies. With a standard of 138.5 kc this corresponds to a range of from 6920 to 13850 kc, extending from the upper edge of the usual audio frequency range, 6920 cps, to short wave radio frequencies as now used in practice. (13850 kc corresponds to wavelength of 21.7 meters.)

V. EXTENSION OF RANGE BY USE OF SECOND MULTIVIBRATOR; LOW FREQUENCIES

While it is relatively easy to attain synchronization of the multivibrator with $m=20$ approximately, it becomes increasingly more difficult as m is made larger. Furthermore, the change in the fundamental frequency of the multivibrator, with respect to the crystal fundamental, becomes less and less as m is made larger, as is easily seen on inspection of (f_c/m). If very low frequencies relative to the crystal fundamental, are desired, it is far more convenient and satisfactory to make use of a second multivibrator, operating on a much lower fundamental than the first. Referring to Fig. 2, the fundamental frequency of multivibrator A is related to the crystal fundamental frequency by f_c/m_a, where the subscript is used to indicate the multivibrator to which the factor m applies. The second multivibrator may be synchronized with the first, so that its fundamental frequency is related to that of the first multivibrator by $(f_c/m_a)/m_b$. The various harmonics of the multivibrator are then related to the reference frequency f_c by $n(f_c/m_a)/m_b$, where n is the order number of the harmonic

with respect to the fundamental frequency of multivibrator B. If $m_a = m_b = m$, it is seen that the fundamental frequency of multivibrator B is equal to the crystal frequency divided by m^2, so that fairly low audio frequencies may be reached from a reference frequency of the order of 150 kc, with values of m which are quite small. For example, with $m = 10$, $f_c = 138.5$ kc, the fundamental of B is 138.5/100 kc = 1385 cps, which is well within the band of usual audio frequencies. Values of $m = 20$ may be easily used, which result in $f_b = 346$ cps, for the same reference frequency as before.*

VI. ARRANGEMENT OF APPARATUS FOR HIGHER RADIO FREQUENCIES

Thus far the discussion has dealt with the production and utilization of frequencies lower than 100 times the reference frequency, where the minimum change in frequency in passing from one harmonic to the next was limited to 1 percent. The range may be considerably extended if smaller variations in frequency are useful.

By utilizing the crystal oscillator circuit of Fig. 5 we are able to greatly accentuate one of the lower order crystal harmonics. The circuit L–C is first adjusted to obtain strong oscillation, as indicated by the maximum of current in the L–C circuit, or by the minimum of dc plate current. Circuit L–n, C–n is then adjusted to resonate at the frequency of the 2, 3, 4 . . . . harmonic of the crystal, as indicated by a maximum of current in this circuit. The adjustment of the L–n, C–n circuit has practically no effect on the behavior of the crystal at fundamental frequency. The adjustment of the L–C circuit, while not

*While it is not at all necessary that m_a be kept equal to m_b it is usually convenient to do so, altering the ratio only when some particular audio frequency is desired. Using the standard previously employed, 138.5 kc, we have for various values of $m_a = m_b = m$, the following audio frequencies for the fundamental of B,

m	f_b cps	m	f_b cps
20	346	12	961
19	395	11	1145
18	427	10	1385
17	479	9	1710
16	541	8	2160
15	615	7	2865
14	707	6	3850
13	819		

Even the restricted values of m_a and m_b here employed result in a large number of audio frequencies, which may be used for comparison of the standard source with an audio frequency standard, or for the calibration of audio frequency sources and other devices in terms of the crystal. The number of unique audio frequencies obtained through the use of the two multivibrators may be greatly increased by varying either m_a or m_b independently.

appreciably affecting the frequency of the crystal, has a marked influence on the magnitude of the harmonic current flowing in the *L–n C–n* circuit. The amplitude of the harmonic current is greatest when the condenser *C* has been reduced slightly from the setting for maximum fundamental current. These changes are plotted in Fig. 6, as functions of the capacity of *C*. The curves show the adjustment of *C*

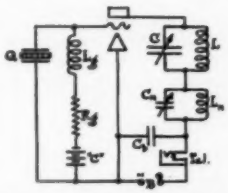

FIG. 5. *Arrangement of circuit for emphasizing one of the low order crystal harmonics.*

for maximum current, of fundamental frequency, in *L–C* to be at a slightly lower value of capacity than that at which the crystal just stops oscillating. The curves I_2 and I_3 rise to a maximum for the same value of *C*, which is 20 to 40 percent lower than the value for maximum I_1, where the subscripts refer to the harmonic in question. The dc plate current reaches a maximum, with variations in *C*, before either

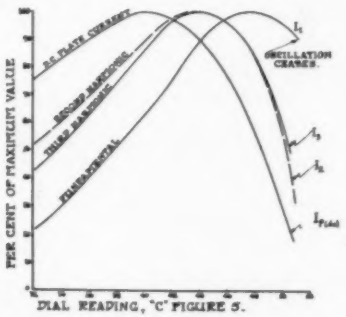

FIG. 6. *Plots of dc plate current, fundamental frequency current, and second or third harmonic frequency current versus values of capacity C, Fig. 5.*

the fundamental or harmonic radio frequency currents. The adjustment for maximum harmonic current is indicated by the plate current having fallen slightly from its maximum value, while the adjustment for maximum fundamental frequency current occurs at a value of dc plate current about equal to the steady current drawn by the tube when the crystal is not oscillating.

The crystal circuit of Fig. 5 may now be arranged for operation with a multivibrator as indicated in Fig. 3, where the multivibrator is coupled to the circuit L–n, C–n and not to the circuit L–C. Two limitations now appear in applying the method. The energy of the

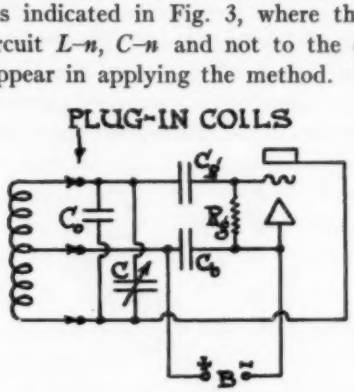

FIG. 7. *Heterodyne oscillator circuit in which changes of frequency due to variations in tube capacitance are minimized.*

harmonics of the crystal is not large so that the synchronizing action of the crystal on the multivibrator is not as good as in the cases which have been previously discussed. Further, the multivibrator fails to function properly when the resistances of the circuit have been reduced

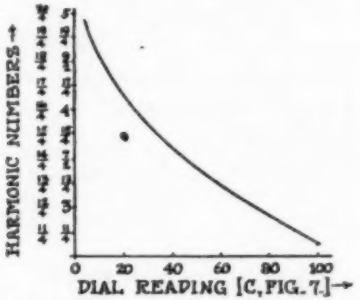

FIG. 8. *Heterodyne calibration in terms of harmonic number versus dial reading, instead of absolute value of frequency versus dial reading.*

to a very few thousand ohms, in an attempt to raise the fundamental frequency of the device. Satisfactory operation is readily attainable for fundamental frequencies up to the order of 750 kc, but the output is quite small. The output is sufficient for calibration work, without the use of special amplifiers, for fundamental frequencies up to the order of 500 kc.

Depending upon the fundamental frequency of the crystal, the first three or four harmonics may possibly be used as outlined above. Even three harmonics provide a tremendous increase in the upper limit of frequency which may be reached, since, if the n_cth harmonic of the crystal is used, the fundamental frequency of the multivibrator is $n_c f_c$, and the nth harmonic of the multivibrator is $n(n_c f_c)$. If we limit outselves to the 100th harmonic of the multivibrator as before, using the second crystal harmonic permits us to go just twice as far on the frequency scale. With the 138.5 kc crystal, the 100th harmonic is 13850 kc; using the second harmonic of the crystal for the multivibrator fundamental, the 100th harmonic becomes 27750 kc, and with the third harmonic it becomes 41550 kc. These last three frequencies correspond to wavelengths of 21.7, 10.8, and 7.2 meters, all of which are sufficiently short to permit observations to be made by means of standing waves on wires within the confines of the laboratory.

Determination of n_c. The value of n_c may be determined by heterodyne methods in a manner analogous to the determination of m. If a series of the crystal harmonic frequencies has been recorded against the dial settings of the heterodyne, it will be found that when listening to the multivibrator harmonics, gaps will be found in the series. The number of crystal harmonics missing between successive harmonics of the multivibrator is equal to $(n_c - 1)$.*

VII. COMPARISON OF AUDIBLE FREQUENCIES WITH VERY HIGH FREQUENCIES

As a final step we may operate three multivibrators, two arranged as already described in Fig. 2 and one as described in Fig. 3, so that a comparison may be made between an audio frequency standard, the crystal standard and short waves on parallel wires, simultaneously.

Reduction of the frequency of the standard to a value suitable for the operation of a small synchronous motor is readily accomplished. The motor may drive a suitable clock mechanism, allowing the fre-

* An analogous example to that given for determining m is given below:

Heterodyne settings for crystal harmonics	Heterodyne setting for multivibrator harmonics	Count of missing harmonics	
98	98	0	
86		1	
49		2	$n_c=3$
35	35	0	

quency of the standard to be compared directly with standard time. This application was carried out at the suggestion of Dr. V. Bush, and will be made the subject of a later paper.

In conclusion the author wishes to express his appreciation for the suggestions and constructive criticisms of the work and of this paper by Prof. V. Bush and Prof. E. L. Bowles, and for the assistance of Mr. Howard A. Chinn in carrying out the laboratory work.

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FEBRUARY 15, 1927.

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A new technique in the use of gratings in spectrography.—The obtaining of x-ray spectra with gratings of 200 lines per millimeter.—The use of a grating having a small number of lines per millimeter (50 to 200) under tangential incidence increases the dispersive power to a considerable degree. A series of spectra are presented of copper and iron in which the lines of the *K* series and their successive orders appear with great clearness. Absolute measurements of wave length are thus possible. The results, in good accord with those obtained with crystalline diffraction, justify Bragg's hypotheses on crystal structure. The method seems to be convenient for the study of those spectral regions to which classic spectroscopy is not applicable, such as the region of very soft x-rays, or the gamma rays of radioactive bodies, or the extreme ultraviolet. Some successful preliminary experiments on the extreme ultraviolet are described.—[Jean Thibaud; Revue d'Optique, *5*, pp. 97–107; 1926.]

G. W. MOFFITT.

A VIBRATION-FREE CAPILLARY ELECTROMETER

By A. S. Gilson, Jr. and G. H. Bishop

ABSTRACT

Description is made of a capillary electrometer designed for physiological studies. The use of this instrument with a thermionic amplifier makes possible the recording of small changes in potential without requiring the construction of an extremely sensitive capillary.

The advantages of the instrument described are the use of a closed acid chamber, an upright position of the capillary, and of even greater importance, the relatively vibration-free meniscus which is obtained by placing the capillary midway between two balancing columns of mercury.

The chief advantages of the capillary electrometer lie in its inexpensiveness and in the accuracy of the records of rapid small changes of potential which may be obtained. Its disadvantages have been so great that at present capillary electrometers are seldom used. The main drawbacks in the use of the instrument are the difficulty of making sufficiently sensitive and even working capillaries, the ease with which the small capillaries become clogged or sticky, the difficulty of eliminating undesirable oscillations of the capillary column as a result of mechanical vibration of the mercury supporting structures, and finally the usual objections associated with the use of sulphuric acid in small open vessels. The first two of these troubles could formerly be overcome only by practice in drawing capillaries and the use of very great care in cleaning the mercury to be introduced into the instrument. The formation of an oxidised film on the mercury surface can be prevented only by the invariable avoidance of continued positivity at the mercury surface or even brief periods of excessive positivity. Elimination of extraneous vibrations has called forth rather elaborate precautions, electrometers having been floated on mercury baths, hung in Julius suspensions, etc.

The improved form of instrument to be described in this paper was designed for use during an experimental study of the action potential developed in the heart with each beat. Induction shocks, which were shunted into the galvanometer circuit, made the use of a string galvanometer inadvisable. The availability of thermionic vacuum tube amplifiers made extreme sensitivity of a capillary unnecessary. The construction of the instrument (Fig. 1*A*) is as follows:

To a horizontal glass tube, *B*, are fused the upright tubes U_1 and U_2. A capillary of proper shape is drawn at the end of the tube *C* which is

then fused into the end of A. The two are then fused into B, midway between U_1 and U_2. This procedure gives three parallel upright tubes opening into B. A glass stopcock is fused to the upper end of A, and leads of platinum wire, $Pt1$ and $Pt2$ are sealed into A and B respectively. All glass parts are of Pyrex.

Rubber tubing attached to the stopcock may be connected with a mercury pressure bulb which is lowered to draw the mercury in C to working level. This level is maintained when the stopcock is closed and movements of the mercury in the pressure bulb or tube do not affect the capillary meniscus. In actual practice, the meniscus is drawn into position by sucking with the mouth, the mercury bulb being dis-

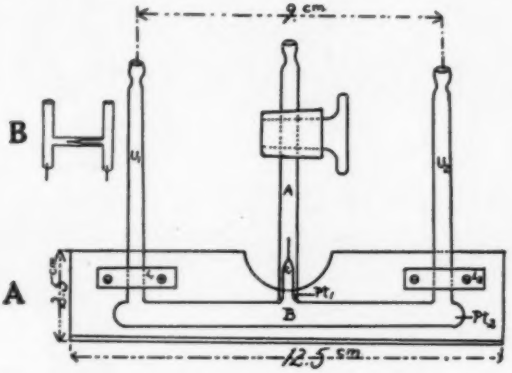

FIG. 1. *A. Drawing showing construction of "vibration-free" capillary electrometer. B. Drawing of one type of capillary electrometer formerly used.*

carded as unnecessary. The support for the electrometer was designed for attaching it to the mechanical stage of a compound microscope carrying a 16 mm objective. The projection distance is 130 cm.

Such a capillary electrometer has proven itself a very satisfactory instrument. It is possible to use a capillary with relatively large bore (around 0.2 mm instead of 0.02 mm inside diameter) and having nearly parallel walls through the working region. If the capillary becomes fouled by the use of excessive voltage, the mercury itself is quickly renewed by drawing a drop of mercury up through the capillary after which the meniscus is easily adjusted again at working level. Accumulated mercury in A is drawn off through a fine glass pipette.

Extraneous vibration is reduced to a minimum by the balancing of the mercury in the open tubes U_1 and U_2 and C. The most accurate

balance is accomplished by making equal the inertia effective upon the surfaces of each of the three upright mercury columns. This is accomplished by placing mercury in U_1 and U_2 to such height that the capillary meniscus at working position is at the same level as the mercury surfaces in the tubes U_1 and U_2. With the mercury level so adjusted, the effect of horizontal vibrations at right angles to the length of B is practically negligible. Vibrations horizontally along B or rotational movements in the plane U_1BU_2 will cause movement of the mercury surfaces in U_1 and U_2 but will cause little or no movement in C because of its mid

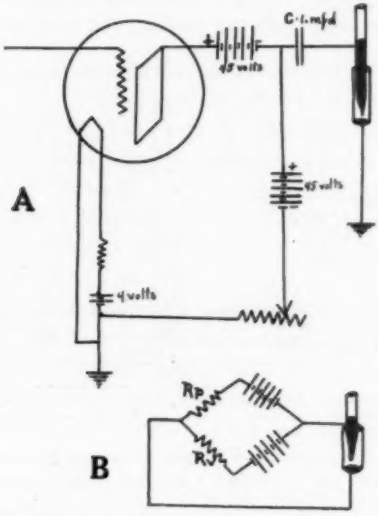

FIG. 2. *A. Diagram of amplifying circuit used with capillary electrometer. B. Diagram indicating the Wheatstone bridge arrangement of the B battery circuit of the amplifier.*

position between U_1 and U_2. Vertical vibrations are reduced to a minimum by virtue of the equal effects of inertia upon the pressure in the three upright tubes.

As now in use in this laboratory, such an electrometer is usually wired with its acid lead attached to a ground wire and the mercury lead joined to the output connection of a one panel capacity coupled thermionic vacuum tube amplifier (Fig. 2*A*) having its B battery circuit balanced, thus making possible the use of an output condenser which need not be perfectly insulated. The action of such a circuit[1] is essentially

[1] K. H. Goode, An improved continuous-reading hydrogen-ion meter. Jour. Am. Chem. Soc., *47*, pp. 2483–2488; 1925.

that of a Wheatstone bridge (Fig. 2*B*) in which the resistance of the plate circuit, R_p, is balanced by the variable resistance R_v, the other two arms of the bridge being balanced by the equal division of the *B* battery voltage. When the circuit is thus balanced, the output condenser, *C*, is at zero potential with respect to the ground and even the use of a rather leaky condenser across the output will result in no drift of the capillary meniscus. Changes of potential at the plate as a result

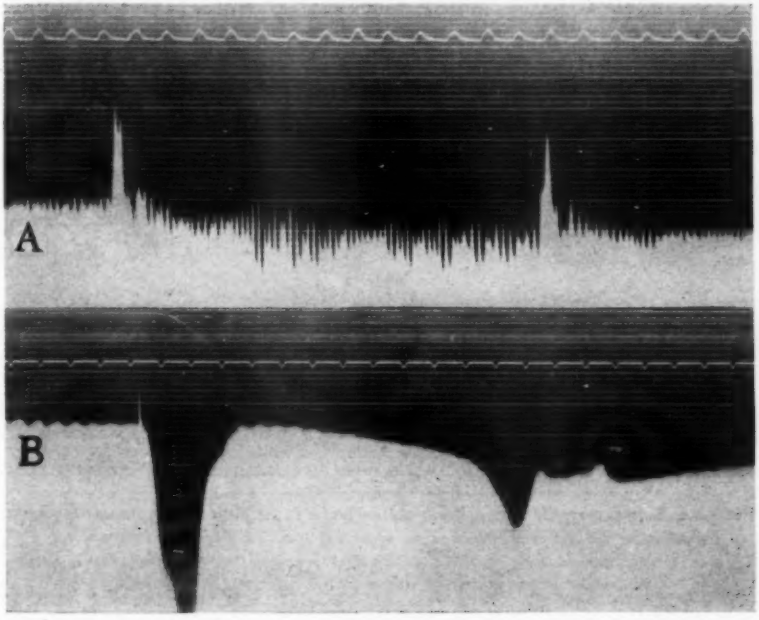

FIG. 3. *Record showing the effect of vibration of the laboratory building upon the capillary meniscus Time intervals = ½ sec. A—Record taken with old style electrometer. B—Record taken with "vibration-free" electrometer. In both the above records, the large excursions of the meniscus were in response to action potentials led from cardiac tissue.*

of changes in the grid potential will result in corresponding changes of potential at *C* and, accordingly, across the capillary.

The apparatus is placed on a table on the fourth floor of a brick and concrete building, situated on a busy street, no precautions to avoid vibration being taken other than those considered in constructing the electrometer. The effect of passing street cars upon our first records made with an electrometer of the form diagrammed in Fig. 1*B* may

be judged from Fig. 3*A*. Fig. 3*B* was taken with the balanced capillary during passage of a street car with an unusually flat tire. The improvement in the legibility of the record is noteworthy.

The records shown in Fig. 4 were made by use of a capillary arranged in this manner. Fig. 4*A* shows the movement of the meniscus under direct application of 44 mv across the capillary. Fig. 4*B* shows the movement resulting from application of a positive voltage to the amplifier input of 8.7 mv, with a falling back of the meniscus due to the slow discharge of the output condenser. In this particular case the amplifier

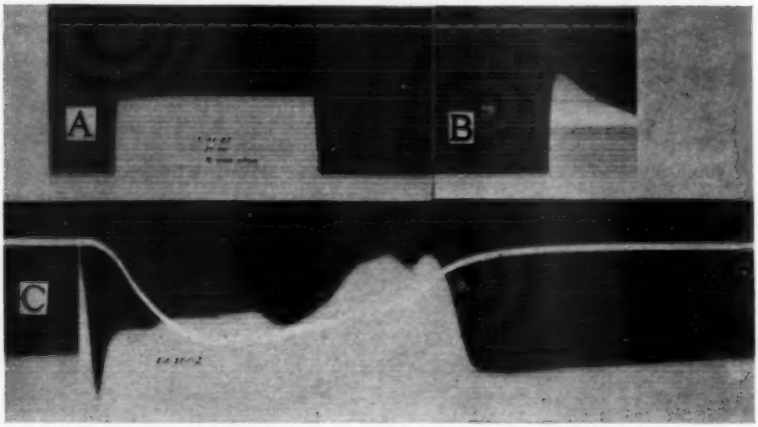

FIG. 4. *A. Normal curve · 34 · mv applied to capillary with 1,000 · ohms series resistance to prevent overshooting. B. Normal curve · 8 · 7 mv applied to grid circuit of amplifier connected to capillary electrometer. C. Typical action potential record obtained from ventricular muscle of tortoise. The white line indicates the mechanical response causing movement of a heart lever.*

was so adjusted as to give voltage amplification of only about five times. Fig. 4*C* is a record of the action potential change following a single effective stimulus, taken by leads from the two ends of a strip of tortoise ventricular muscle. The time record in these figures shows one-fifth second intervals. Records obtained by the use of the capillary with the amplifier may be corrected by methods similar to that used for records made with the capillary electrometer alone.

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A DEVICE TO DRAW VACUUM TUBE CHARACTERISTIC CURVES AUTOMATICALLY

By G. C. Campbell and G. W. Willard

A device for automatically drawing the characteristic curves of vacuum tubes more rapidly and accurately than by plotting points, unless more than usual care is taken, can be arranged rather easily from the apparatus usually available in the laboratory.

The usual circuit for obtaining characteristic curves was used (Fig. 1). The actual recording was done by a Leeds and Northrup recording pyrometer of the potentiometer type which is designed to draw the curve of potential difference across thermocouples—the potential

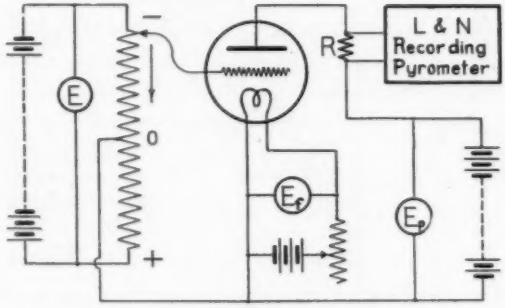

Fig. 1. *Circuit for obtaining characteristic curves.*

difference being the ordinate, time the abscissa. For our purpose, it must plot current against potential. To make the ordinate represent plate-current, it is necessary only to put a known resistance of suitable magnitude in the plate circuit and connect the recording pyrometer across it. The instrument thus records the potential drop across the resistance (R), which is of course directly proportional to the current. To make the abscissa represent the grid-potential, the roll of coordinate paper must be rotated and the grid-potential varied uniformly together. This requirement is taken care of by operating both the paper roll and the grid-potential potentiometer by the same motor through speed reducing gears.

We first tried an ordinary slider rheostat potentiometer to vary the grid-potential. The slide was connected by a string to the speed

reducing gears. It was found, however, to give a rather irregular variation of the potential because of imperfect contact as the slider was moved. It would be satisfactory for rough work. We eventually used a W. G. Pye drum potentiometer with a tap taken off internally at exactly the center of the coil in order to locate the zero grid-potential at the center of the potential interval used. The same purpose might be accomplished by taking a tap off the center of the battery—but not so accurately. The usual crank on the potentiometer was replaced by a pulley which was belted to the speed reducing gears.

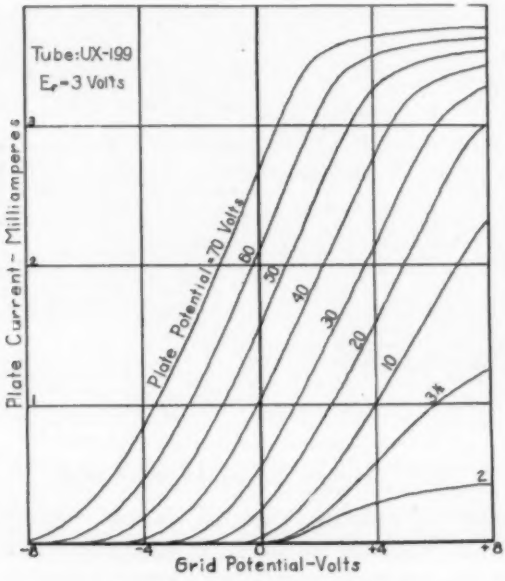

FIG. 2. *Characteristic curve for UX-199 tube.*

The recording instrument has a range of 20 or 60 millivolts for full scale deflection,—the width of the coordinate paper which is ten inches. The resistance R should be of proper magnitude to give sufficient potential drop for nearly full scale deflection at the maximum plate current. The curve is started at the maximum value of the grid-potential—otherwise it would be opposite to the conventional form.

Fig. 2 shows a sample family of characteristic curves for a UX-199 tube drawn by this apparatus. By proper adjustment of the potential across the grid potentiometer, the potential interval was made to

coincide with a multiple of the unit of the coordinate paper. It may be seen that the curves are composed of minute steps, caused by the potential drop across R being balanced against the potentiometer circuit of the recorder at two second intervals,—the pen remaining at constant level within the interval. This fact sets the low time limit for drawing a satisfactory curve at about five minutes. Such a curve would contain 150 steps. Satisfactory curves can be obtained of currents so small that the instrument may be adjusted to give full scale deflection for a maximum current of 0.5 milliampere. Obviously, we must increase the value of R in order to give sufficient potential drop for full scale deflection at low current values. This decreases the sensitivity of the galvanometer in the recording pyrometer and fixes the low current limit for satisfactory operation with any given galvanometer in the instrument.

The usefulness of the device for research purposes would be materially increased if the instrument maker could supply a similar recording instrument with a more sensitive galvanometer, and a faster balancing mechanism.

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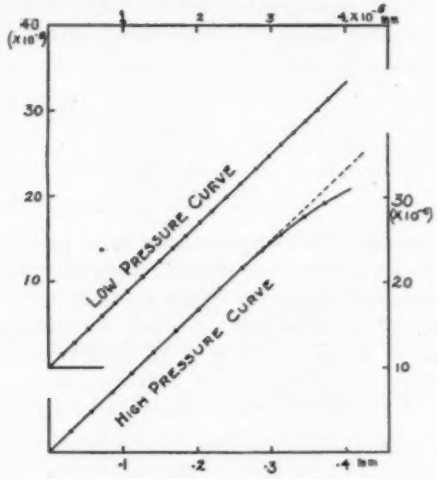
An absorbant wedge optical pyrometer and micropyrometer.—The author describes a modification of the Féry pyrometer. The current in the filament of the pyrometer lamp is held constant while the filament is made to disappear by the adjustment of a pair of absorbing glass wedges. A micropyrometer is obtained by using an objective of comparatively short focal length. The uncertainty of a determination of temperature does not exceed 1° at 2000°C. Various arrangements are proposed for maintaining a filament brightness constant within the required limit of half of one per cent. The method of constant filament resistance allows considerably higher precision of brightness control. It also makes possible the arrangement of a simple control bridge so designed that outside temperatures have no influence on the brightness of the filament.—[G. Ribaud; Revue d'Optique, *5*, pp. 289–327; 1926.]

G. W. MOFFITT

A HOT WIRE VACUUM GAUGE

By A. M. Skellett

In an investigation on dissociation in hydrogen by electron impact, it was found that an ionization gauge was unsatisfactory as a pressure indicator, as it itself cleaned up the gas to a certain extent. The gauge finally used was a modified form of a Pirani hot wire gauge, the outstanding advantages of which were first, that the pressure-galvanometer reading curve was a straight line over the range of the pressure used, and second, that individual settings were not necessary for each reading allowing the changes in pressure to be automatically plotted against the time as a curve. This is a distinct advantage over the bridge method formerly used with this type of a gauge.[1]



Range of Pressures Used in Calibration

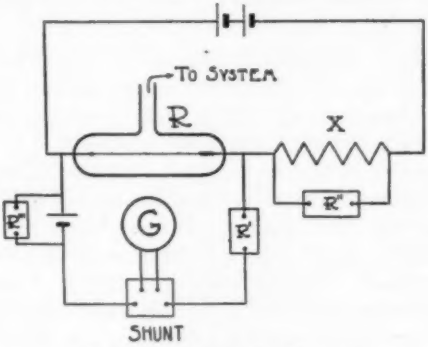
A filament of one mil tungsten wire about 5 cm long was mounted along the axis of a pyrex tube about 1 cm in diameter. The filament was kept taut by means of a small spring at one end. The tube was set up in a thermostat as it was very sensitive to heat changes and the resistance X was made of wire with a thermal resistant coefficient of

[1] L. Dunoyer, "Vacuum Practice," Eng. Ed., p. 92. King, Proc. Phys. Soc. of London. Cox, J. O. S. A. & R. S. I. 9, p. 569; 1924.

practically zero. R'' was a fine adjustment resistance used in balancing the circuits and R' along with the shunt protected the galvanometer and governed the sensitivity.

The filament was connected in series with a 4 volt storage battery and the resistance X, of about the same value as the resistance of the filament, and the potential drop across the filament balanced by means of a subsidiary battery of 2 volts in series with the resistance R' and the galvanometer. The balance was adjusted with the highest obtainable vacuum.

Now if a little gas was admitted, a certain amount of heat was conducted away from the filament by this gas and the filament becoming cooler changed resistance thereby unbalancing the current in the subsidiary circuit and affecting the galvanometer. It was found that



Electrical system used with gauge.

the galvanometer readings were directly proportional to the pressures up to .2 mm of mercury. The accompanying curves show the range over which the galvanometer readings were directly proportional to the pressures.

It is well known that McLeod gauge readings are inaccurate at low pressures, i.e., near the top of the scale, and so in order to calibrate the gauge, gas was allowed to run in through a fine capillary from a reservoir. Making the justifiable assumption that the amount of gas accumulated is proportional to the length of time during which the gas has been flowing through the capillary, it was possible to determine the pressure at any time during the process, if the pressure accumulated at the end of the time was measured by a McLeod gauge.

We believe that the lower limit of this gauge is restricted only by the constancy of the voltages of the two batteries and the changes in these

can be made equal (thus not disturbing the zero point) by regulating the amount of current flowing from the 2 volt battery by means of the resistance R'''. Using this gauge pressures as low as 6×10^{-6} mm were accurately measured and the indications were that with a more sensitive galvanometer the lower limit might have been pushed still farther.

In conclusion the writer wishes to express his thanks to Professor A. L. Hughes, at whose suggestion this problem was carried out, for his aid and interest.

WASHINGTON UNIVERSITY,
ST. LOUIS, MISSOURI.

Makers of Science: Electricity and Magnetism. By D. M. Turner. xv+179 pages. Oxford University Press (American Branch, 35 W. 32d St., New York). $2.50.

This little volume is addressed "to the young student and to the general reader interested in science," but it will appeal also to the more mature student, who will find condensed in the book a wealth of biographical and historical information not otherwise easily accessible.

The subject matter is divided into ten chapters. Chapter I, "Early Speculations," discusses the work of the ancients and of Gilbert and brings the subject up to the 18th century. Chapter II, "Electrostatics in the Eighteenth Century," devotes special attention to the work of Franklin, Priestly and Cavendish. Then follows "The Electric Current" (Chapter III) and "Electromagnetism" (Chapter IV), describing developments in these subjects from Galvani to Ohm. Chapter V is devoted to the work of Faraday. The development of electrical measurements and units during the middle half of the nineteenth century is discussed in Chapter VI, while Chapter VII presents the development of Maxwell's Electromagnetic Theory. "Electrolysis," from Faraday to the close of the nineteenth century, occupies Chapter VIII. That fascinating subject, the discharge of electricity through rarefied gases, is traced in Chapter IX, from the observations of Hauksbee in the early eighteenth century, to the discovery of the electron by Thomson in 1897. The concluding chapter makes a few brief comments on twentieth century problems.

The book is well illustrated by numerous diagrams. It contains brief, but interesting, biographical material concerning many of the leading scientists whose work is discussed. And it is well indexed. Its brevity, in places disappointing, is of course enforced by space limitations. Although numerous references are given, the book would have been improved by a collected and classified bibliography.

F. K. RICHTMYER

A LOW PRESSURE VALVE

By Melville F. Peters and Ernest H. Klein

Two types of valves to regulate the flow of a gas under reduced pressure are in use, first, the familiar finely drawn capillary tubing and second, mechanical valves. The capillary tubing frequently proves unsatisfactory because of its frailty, tendency to become clogged, and total lack of control. The mechanical valve, because of its complexity, offers numerous difficulties. The following apparatus has been devised to secure a sturdy, compact valve that will offer, with the greatest simplicity, close regulation over an appreciable range of pressures.

A lead solder is melted, under vacuum to prevent oxidation, into a glass U-tube. Upon cooling a small opening exists between the glass and solder surfaces. By taking advantage of the comparatively wide

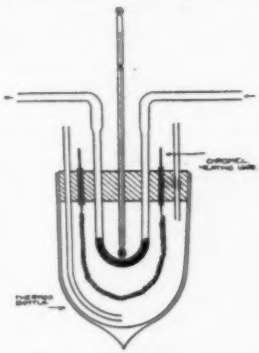


Fig. 1. *Low pressure valve.*

difference in the coefficients of expansion of the glass and the solder, this opening may be varied, thus securing a corresponding variation in the volume of gas dragged through the U-tube and, as a consequence, in the pressure of the system. To maintain the valve at a desired temperature, the U-tube and a thermometer are immersed in a thermos bottle. The thermos bottle is equipped with a chromel heating unit and a means of admitting and removing cooling solutions. The rate of flow of the gas is a function of both the length of the solder and the diameter of the tubing, so, by varying either or both of these factors, valves may be constructed that will function over different pressure ranges.

Using a U of 3mm Pyrex tubing containing 4 cm of solder, to control the supply of hydrogen to a certain low pressure system, the following control was secured:

Temperature of valve	Pressure of gas in system	Volume of gas admitted (liters per hour)
26.0°C	0.57 mm	0.11
37.8	0.51	
46.3	0.47	
51.6	0.43	
62.8	0.38	
71.0	0.34	
81.5	0.30	
90.2	0.26	0.01

As is seen, the pressure varied practically uniformly with the temperature, and further calibration, within the range of experimental error, checked the first values, indicating that there was no change in

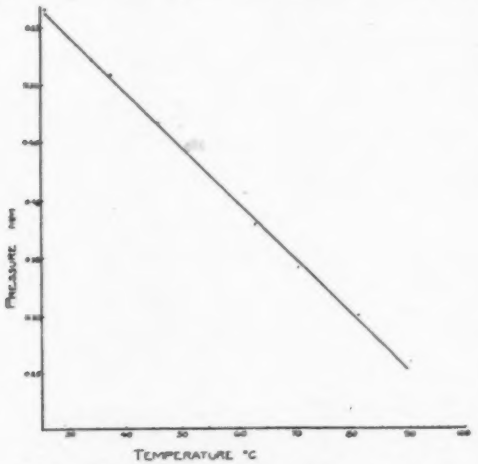


FIG. 2. *Variation of pressure with temperature of valve.*

the valve after having been submitted to a variation in temperature. In addition to use with hydrogen and oxygen, the valve has been used for a week admitting air from a dusty room without becoming stopped. In case of an obstruction, it is only necessary to equalize the pressure on both sides of the valve and melt the solder. The foreign matter is adsorbed by the solder and, upon cooling, the valve functions as

before. By the use of capillary tubing (1 mm diameter) very low pressures may be obtained.

Valves may be produced by taking advantage of the difference in coefficients of expansion of other substances. Solder was used in this instance because of its low melting point, ductility, and relatively high coefficient of expansion.

NEW JERSEY ZINC CO. RESEARCH LABORATORY,
PALMERTON, PENNSYLVANIA

Thermal Expansion of Graphite. This paper gives the results of an investigation on the thermal expansion of longitudinal and transverse sections of artificial graphite electrode (99. to 99.7 per cent carbon) over various temperature ranges between room temperature and 600°C.

A summary of available data by previous observers on the thermal expansion of graphite (natural and artificial) and of other forms of carbon is included.

In the present investigation the coefficients of expansion of graphite were found to be low. The transverse samples expand considerably more than the longitudinal samples (approximately 45 per cent). For both the longitudinal and transverse samples the coefficients of expansion decrease slightly as the purity (carbon content) increases. The table gives a résumé of average coefficients of expansion derived from the data on all samples for various temperature ranges between 20° and 600°C.

Material	Average coefficients of expansion per degree Centigrade					
	20 to 100°C	20 to 200°C	20 to 300°C	20 to 400°C	20 to 500°C	20 to 600°C
	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$
Longitudinal	1.9	2.0	2.2	2.4	2.5	2.7
Transverse	2.9	3.0	3.2	3.5	3.6	3.7

The material here reported on was obtained from the Acheson Graphite Company of Niagara Falls. On subjection to x-ray crystal analysis a diffraction pattern of intense definitely spaced bands clearly indicated crystalline structure. The difference in the expansion between the specimens cut transversely and longitudinally are thought to be due to the process of manufacture in which the compression is different in the two directions. [Bureau of Standards Technological Paper No. 335 by Peter Hidnert and W. T. Sweeney, published February 18, 1927.]

C. C. BIDWELL

THE USE OF THE WIRE METHOD FOR THE MELTING POINT OF PALLADIUM

By C. E. Mendenhall and C. L. Utterback

Everyone who has used the so called "wire method" for determining the melting point of palladium has doubtless noticed the unfortunate characteristics of this metal which make it much more difficult to obtain as consistent results with it as can be obtained with gold used in the same manner. The practice in this laboratory has been to connect the palladium, in the form of a short length of fine wire or narrow thin strip, to two fine insulated platinum wires leading out of the furnace, the whole forming part of an electric circuit so arranged that the melting of the palladium breaks the circuit. The instant of melting is thus indicated on some suitable instrument. Under these circumstances several things may happen which will prevent the breaking of the circuit until the temperature of the furnace has risen considerably above the melting point of palladium: (a) the palladium may settle

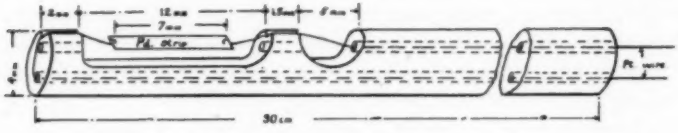


Fig. 1.

into very intimate contact with the porcelain tube used to support it, and thus maintain the circuit even though melted, or, (b) the palladium may draw together into a bead pulling the platinum wires into contact, or, (c) there may be formed an alloy of higher melting point by diffusion of the platinum into the palladium from the points of contact with the platinum leads. Such diffusion occurs with surprising rapidity if, as might appear desirable for other reasons, the palladium is electrically welded to the platinum leads.

The arrangement shown in the accompanying diagram has been found to avoid these difficulties more successfully than any other of the numerous forms which have been tried. The construction will be clear without detailed description, but it should be remarked that the indicated length of the palladium strip is the minimum which has been found safe. The other dimensions were chosen as small as possible

because of the small size of the black-body furnace in which the arrangement had to be used.

For the delicate tubes of highly refractory porcelain the authors are indebted to the kindness of Dr. F. H. Riddle, director of research of the Champion Porcelain Company.

The following set of individual observations of the melting point of palladium indicate the degree of concordance which may be obtained.

Observation. (sample *K*)	Potentiometer reading in millivolts	Difference from the mean in degrees Centigrade
1	16.030	0.08
2	16.026	0.25
3	16.027	0.16
4	16.034	0.41
5	16.032	0.25

This seems to us very satisfactory, and is certainly much better than we have previously been able to secure.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF WISCONSIN,
JUNE 3, 1926.

On the spontaneous reversal of neon lines.—Most of the red lines of neon show a self reversal when one observes the light emitted by a thickness of several centimeters of the gas. This reversal is easily observed and studied by means of a Fabry and Perot interferometer in combination with a spectrograph. One may thus study several types of reversal and obtain the distribution of intensity across the width of the line. A comparison of the relative intensities in a line observed in a capillary tube along the axis and normal thereto indicates an absorption of the middle portion of the line, and not a double line. Consequently these lines can not be substituted for the red line of cadmium as fundamental standards of wave length.—[H. Buisson and C. Jausseran; Revue d'Optique, *5*, pp. 149–160; 1926.]

G. W. MOFFITT

A CONVENIENT CHART SHOWING RESISTANCE OF WIRES

By W. H. Stannard*

The writer has had to make many calculations involving the resistance of wires of various sizes and of various materials, and has also been called upon many times for such information by other members of the organization with which he is connected. The data are commonly published in the form of wire tables. A separate table is published for each material; some tables are in books, others on single sheets, and all are different in form, arrangement and size.

The accompanying plate assembles the information given in numerous wire tables in a convenient and easily read chart. It was developed from a similar chart that the writer drew up several years ago and has used ever since with complete satisfaction.

Each line sloping down to the right is in effect a complete wire table for a particular material, except that the diameter and area are given by the lines sloping up to the right. The weight of wire is seldom required and then only approximately; therefore, a single line only, corresponding to the weight of copper, is given. The area, diameter, and weight in the units stated are to be read on the resistance scale with no shift of the decimal point.

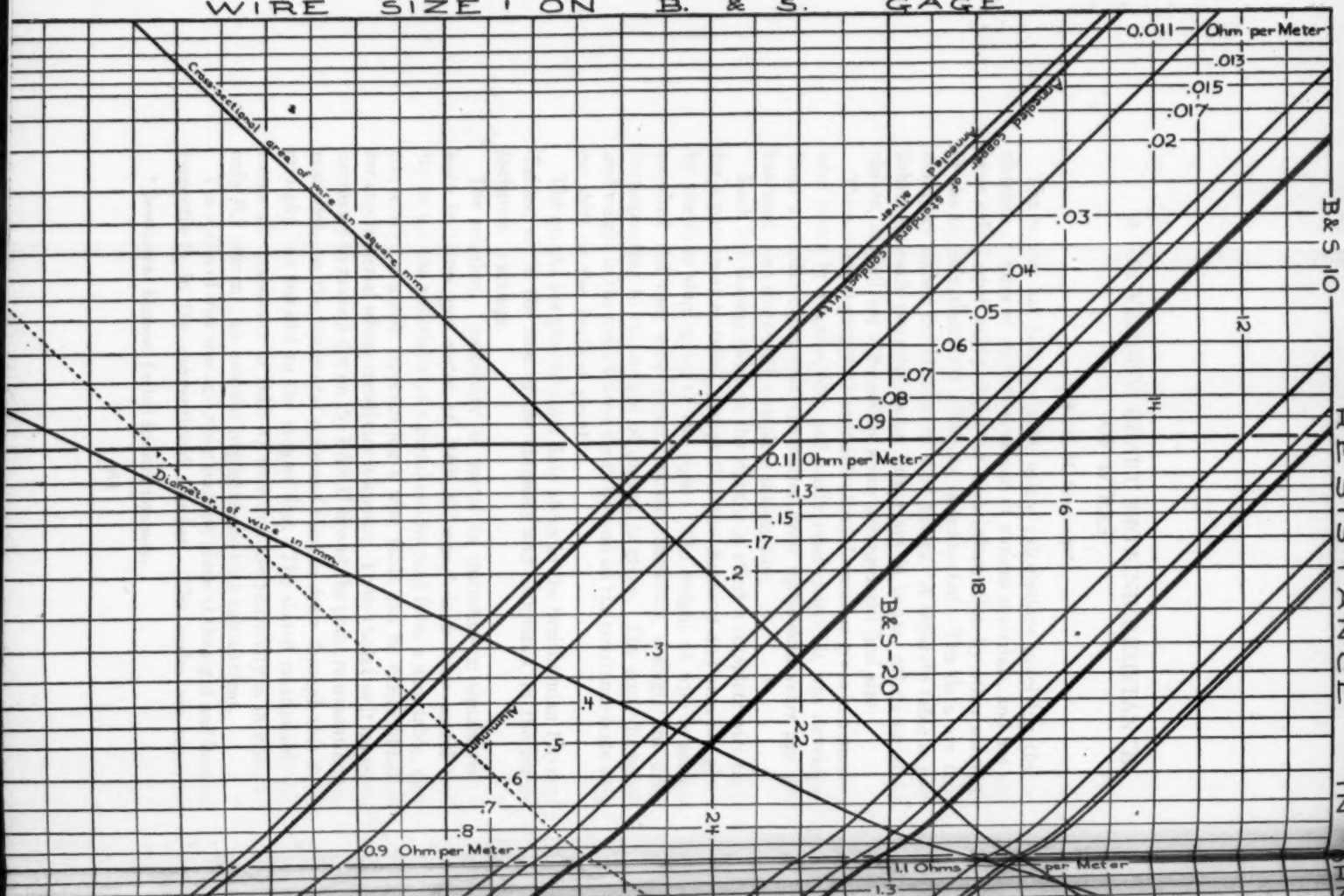
The graphs are plotted from data given in the Smithsonian Physical Tables, or, in the case of the chromels and nichromes, in the manufacturers' catalogs.

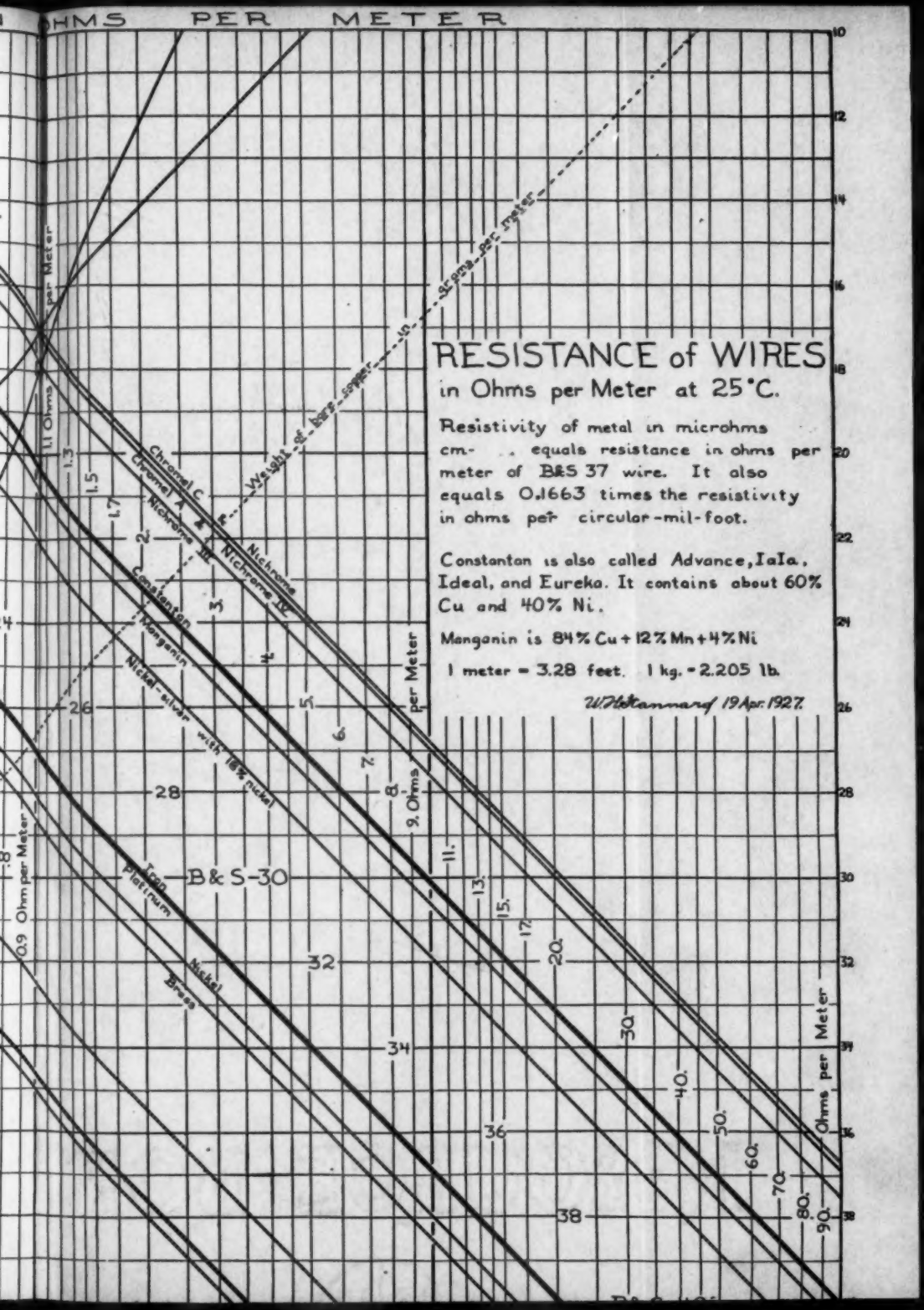
The relation, "resistivity of metal in microhm-cm equals resistance in ohms per meter of B&S 37 wire," is a very useful one. By its use the resistivity of a metal can be read from a wire table, and, conversely, a graph representing a wire table can be easily obtained for any material whose resistivity is known. To do the latter it is merely necessary to make a dot on the B&S 37 line at the point representing the resistivity on the horizontal scale and then to draw through this dot a straight line parallel to the copper line. The stated relationship between the resistance of B&S 37 wire and the resistivity is in error by only 0.3 percent, an amount negligible in wire calculations.

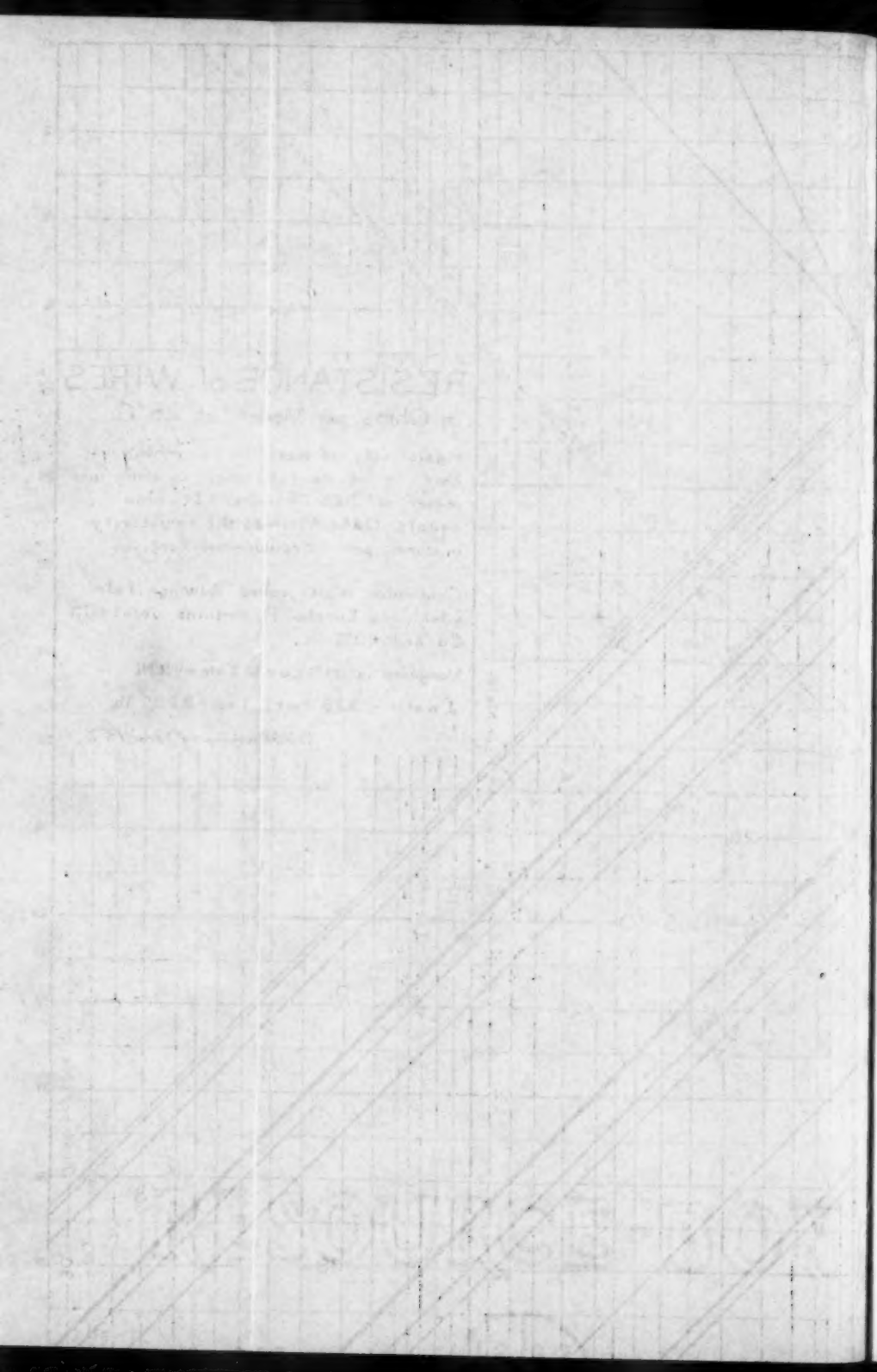
The scales of wire size and resistance are given at the right and bottom respectively in the conventional manner. The scales are repeated

* Development Engineer, Central Scientific Company.

RESISTANCE
B.& S. GAGE
WIRE SIZE - ONE


WIRE SIZE ! ON B. & S. GAGE
Cross-sectional area of wire in square mm.
Diameter of wire in mm.
Aluminum
Annealed copper of standard conductivity
annealed
0.011 Ohm per Meter
.013
.015
.017
.02
.03
.04
.05
.06
.07
.08
.09
0.11 Ohm per Meter
.13
.15
.17
.2
.3
.4
.5
.6
.7
.8
0.9 Ohm per Meter
1.1 Ohms per Meter
1.3
B&S 10
12
14
16
18
B&S-20
22
24
RESISTANCE IN


OHMS PER METER
RESISTANCE of WIRES
in Ohms per Meter at 25°C.
Resistivity of metal in microhms cm- . . equals resistance in ohms per meter of B&S 37 wire. It also equals 0.1663 times the resistivity in ohms per circular-mil-foot.
Constantan is also called Advance, Ia Ia., Ideal, and Eureka. It contains about 60% Cu and 40% Ni.
Manganin is 84% Cu + 12% Mn + 4% Ni
1 meter = 3.28 feet. 1 kg. = 2.205 lb.
W Hoskannard/ 19 Apr 1927
Weight of bare copper in grams per meter
Chromel C
Chromel A
Nichrome
Nichrome III
Nichrome IV
Constantan
Manganin
Nickel-silver with 18% nickel
Iron
platinum
Nickel
Brass
B & S-30
11 Ohms per Meter
1.3
1.5
1.7
2
3
4
5
6
7
8
9. Ohms per Meter
11.
13.
15.
17.
20.
30.
40.
50.
60.
70.
80.
90.
Ohms per Meter
0.9 Ohm per Meter
26
28
30
32
34
36
38



in the body of the chart in the form of numbers placed as closely as practicable to the places that will be read most frequently. The writer has found these central scales more conducive to speed and accuracy than the conventional border scales.

The metric system only is used because it is the convenient one. Even in cases where the requirements are given and the results must be stated in English measure, the writer has found that in making calculations (of an electromagnet or instrument coil, for example) it saves time and prevents mistakes to convert all the initial data into metric units at the start and to convert the results into English units at the end, rather than make any calculations whatever in the cumbersome English system.

DEVELOPMENT LABORATORY,
CENTRAL SCIENTIFIC CO.,
CHICAGO, ILL.

Vacuum Practice. By L. Dunoyer. Translated by J. H. Smith, x+228 pages. D. Van Nostrand Company, New York. $4.00.

The French edition of this book appeared in 1924 under the title, La Technique de Vide. The translation follows the original, with one or two minor additions.

The book is divided into four chapters. Chapter I is devoted to pumps. Among those described are: The Geissler pump; the Sprengel pumps; various piston and rotary oil pumps; the Gaede rotary pump; the Gaede diffusion pump; and the Langmuir "condensation" pump, in its various forms, single and double stage. Data is given showing operating characteristics.

Chapter II discusses the very important subject of manometers, including their theory and operating characteristics. The treatment includes the McLeod Gauge, the absolute manometer, radiometer gauges, thermal conductivity gauges, the ionization gauge and viscosity gauges.

The third chapter, mainly on technique, will be appreciated by everyone working with high vacuua. An extensive treatment is given of connecting tubes, with many practical hints of construction. Some twenty pages are devoted to the elimination of occluded gases. Special attention is given to the micro analysis of gases.

Chapter IV takes up the production and improvement of vacuua by absorbers such as charcoal; and palladium black (for hydrogen) and by the electric discharge; i. e., the so-called "clean up" effect.

A classified bibliography is also included.

The increasing use of high vacuua, both in research and in industry, makes this book a timely addition to the literature of the subject.

F. K. R.

A POLARIMETER FOR DEMONSTRATING THE LAW OF MALUS

By LeRoy D. Weld

In 1808 Malus enunciated the well known law, according to which the intensity of a beam of light, which has been successively reflected at the polarizing angle by each of two glass surfaces in planes of reflection making a variable angle ϕ with each other, varies in accordance with the relation

$$I = I_0 \cos^2 \phi. \tag{1}$$

The testing of this law affords the laboratory student in optics one of the most elegant experimental proofs of the validity of the polarization theory. The apparatus herein described is easily constructed in the laboratory shop, and has proved entirely satisfactory.

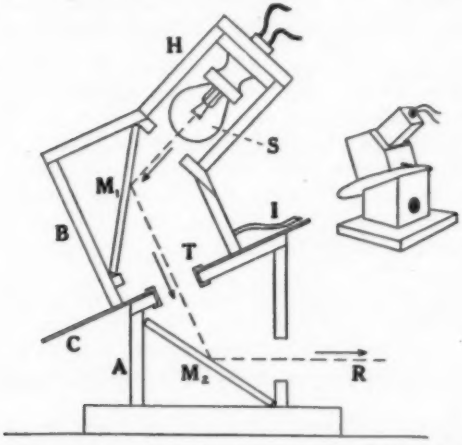


Fig. 1. *Section of the reflection polarimeter. The instrument is 27 cm in height.*

The reflection polarimeter itself (Fig. 1) consists of a fixed, box-like chamber A, upon the sloping "roof" of which a second box B rotates about the tubular axis T. The boxes contain mirrors M_1, M_2, of black glass. Opening out of B and attached rigidly to it is the closed lamphouse H, which contains a small but brilliant lamp S (a 12-volt automobile spotlight bulb), used as the source of light. The rays from this lamp are reflected at the polarizing angle of about 57° by the mirrors

M_1 and M_2 in succession, and finally emerge as a horizontal beam R through an opening in the front of the box A. Between the boxes A and B is a fixed circular brass plate C, the upper surface of which is graduated into degrees. B is provided with an index I, which moves over this scale, thus giving the angle ϕ between the two planes of reflection SM_1M_2 and M_1M_2R.

As the box B is turned about T, carrying the lamp S with it, the intensity of the emergent beam R varies in accordance with Malus' law.

This apparatus is mounted conveniently at the end of an ordinary photometer bench. The box housing the photometer screen remains fixed at the zero of the scale. The lamp which, moving along the scale, serves for the intensity comparison standard, should be of low power, say one cp, and may conveniently be a small electric bulb controlled by a rheostat, or it may be a small gas jet. The angle ϕ on the polarimeter circle is first set at zero, so that the intensity of the beam R

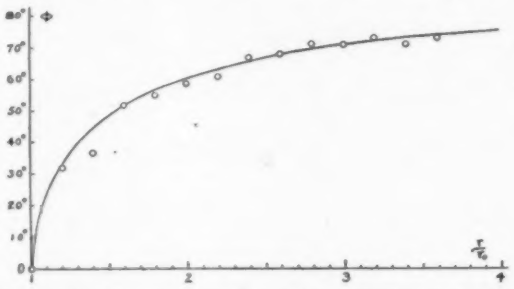


FIG. 2. *Typical results. The smooth curve corresponds to the theoretical law of Malus.*

is I_0, and the photometer is balanced with the comparison lamp at some initial distance r_0 from the screen. By first adjusting the brilliance of the comparison lamp, the initial distance r_0 may be conveniently chosen and the photometer readings r may be kept within the range of the scale until ϕ is nearly 90°.

It is found most practicable to vary r by steps and to adjust ϕ accordingly in securing the photometric balance. The observed values of ϕ may then be plotted in terms of the ratio r/r_0. According to the law of Malus, the result should be the secant or reciprocal cosine curve:

$$1/\cos\phi = r/r_0.$$

A series of results of a typical set of single readings taken with this apparatus, using a small gas jet as the comparison lamp, is shown in

Fig. 2. The plotted points indicate the observed results; the smooth curve corresponds to the theoretical values of ϕ deduced from (2) above.

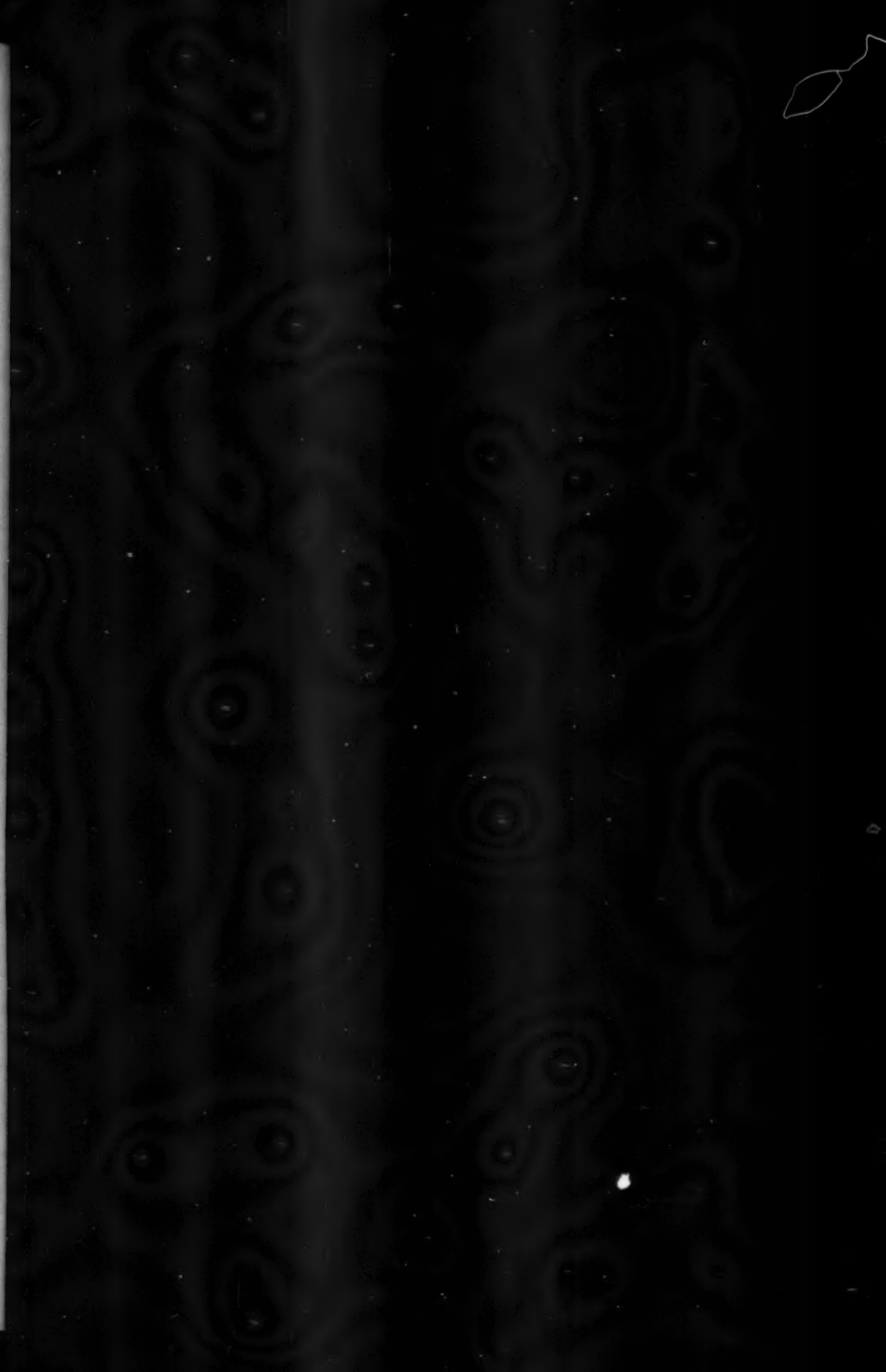
The polarimeter here described was constructed in the Coe College laboratory shop by Mr. Leonard Albright.

COE COLLEGE,
CEDAR RAPIDS, IOWA.

Patents—Roger Sherman Hoar, Head of Patent Department of Bucyrus Company. vi+224 pp. Published by the Ronald Press Company, New York City. $4.50.

It is generally admitted that patent practice is a highly specialized branch of law. Some few executives consider that they know all about it and are mistaken; many others consider it a very complex subject on which it is hopeless for them to become informed and they also are mistaken.

This book on "Patents" is written primarily for the business executive and in it the author has indicated the complexity of the subject, with the need for expert advice, and at the same time has explained in a very readable manner the fundamentals of patent law and practice. One might desire to see a few more references (possibly as foot notes) to court decisions thus affording a point of departure for those who wish to go further into some matter of interest. Also, there is room for improvement in the explanation of patent claims or their meaning, and it would probably be helpful if such explanation came earlier in the book; chapter 6 might well precede chapter 3. Putting aside these minor considerations, however, it may be said that the book is remarkably free from the cumbersome style so characteristic of writers in this field. Certain of the chapters stand out as excellent; chapter 3 on "Anticipation and Domination" is very satisfactory; chapters 7, 11 and 13 on "Patent Office Tactics," "Choice of Attorney" and "Searches," respectively, are filled with sound and suggestive advice and are amply sufficient in themselves to justify the book.

The book could be read with profit by all who are interested but not acquainted with patent matters, and will be enjoyed and appreciated most by those who have specialized in this field. Its best recommendation, however, is that it is very well written for the audience it is intended to reach.

P. I. WOLD